INTEREST RATE BENCHMARKING

A TRANSFER PRICING GUIDE

PART II

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List of Abbreviations

The following abbreviations and symbols are used in this guide:

AAF	Annuity adjustment factor
ABL	Asset-backed lending
ac.finance.IRB	Excel/java-based interest rate benchmarking tool developed as part of this guide
ATSM	Affine term structure model
CNS	Comparable note search
CRA	Credit rating analysis
CUP	Comparable unrelated price
CUT	Comparable unrelated transaction
DCA	Debt capacity assessment
DRB	Discount rate benchmarking
EMTN	Euro Medium Term Note
FMV	Fair market value
FTE	Flow-through entity
FX	Forward exchange
ICS	Internal CUT search
IQR	Interquartile range
IRB	Interest rate benchmarking
LBMA	London Bullet Market Association
MNE	Multinational enterprise
MTN	Medium Term Note
MYCA	Market yield curve analysis
NPV	Net present value
NSM	Nelson-Siegel model
OECD Guidelines	"BEPS Actions 8 – 10, Financial Transactions", a draft published in July – September 2018 for the purposes of public discussion
OID	Original issue discount
OID Note	Original Issue Discount Note
OLS	Ordinary least square
PIK	Pay-in-kind
PLOI	Pertinent loan or indebtedness
ROE	Return on equity
TP	Transfer pricing
VAR	Vector auto-regression

Section 1 Introduction

Transfer pricing.

Section 2 Static Models of Interest Rates

The IRB analysis can be generally broken down into two parts:

- ► Specification and estimation of the interest rate model including estimation of risk and term premiums;
- ► Adjustment of the loan interest rate for differences in options, provisions, currency, and interest payment terms.

In this section we present alternative models of interest rate and discuss the models' estimation methods. In the next section we discuss briefly how each adjustment for a loan interest is performed.

2.1 Basic regression model

Basic model assumes the following statistical equation applied for the interest benchmarking analysis. Suppose that $\{y_i\}$ is a sample of yield rates identified based on a search procedure. The yield rates are assumed to be bullet rates. Suppose that the credit rating and maturity term of the underlying CUT i are denoted respectively as cr_i and t_i. Suppose that the sample of yields is ordered by respective CUTs credit ratings cr_i and for each specific credit rating the yields are further ordered by the maturity terms t_i.

(2.1) $$y^g_{i,r,\tau} = \pi^g + \pi_{r,\tau} + \sigma^g \times \varepsilon_i$$

where parameters of the model are interpreted as follows:

(i) Index $g = 0{,}1$ corresponds to two groups: $g = 0$ is the broad sectoral group and $g = 1$ is the narrow sectoral group;

(ii) Parameter $\pi^0 = 0$ and $\pi^1 = \pi$ models the premium in the narrow sectoral group;

(iii) Parameters $\pi_{r,\tau}$ model term and risk premiums, where r is the credit rating of the respective transaction and τ is the remaining term to maturity;

(iv) Parameter σ^g models the variation in the yield rates, which differs between the narrow and broad industry sectors but does not depend on credit rating or remaining maturity term of the transaction.

In the model specification, the estimation of the parameters is effectively broken down into two steps:

1. Estimate $\pi_{r,\tau}$ from the broad sectoral sample of yields. In practice, the parameters are estimated using the broad sectoral curves estimated by Blomberg or Reuters.
2. Perform interest benchmarking analysis to estimate the narrow sector premium π^g and volatility parameter σ^g.

The MYCA analysis effectively assumes that sector premium is zero $\pi = 0$ and the parameters $\pi_{r,\tau}$ estimated for the broad sector using Bloomberg or Reuters standardized yield curves produce an unbiased estimate of the expected yield rate on the tested transaction. Under the CNS approach, the customized search parameters are applied to test whether it is reasonable to assume that parameter π is zero. If both the CNS and MYCA approaches have a common range, then the common range is reported and interpreted as no material sector-specific premium is present.

Under the ICS approach, parameter π can be interpreted as the premium, which is specific to the parent group of the tested entity (which may include the sector-specific and other parent group idiosyncratic risks). For the Canadian transfer pricing analysis, ICS is normally a preferred approach as it is the most accurate modelling approach to capture group-specific risks. However, it is generally preferred if both IXCS and CNS have a common range so that the factors that determine the price are interpreted as normal market factors: (i) term premium, (ii) risk premium; and (iii) sector and/or country premium. Presence of the common range produces more robust interest benchmarking results and shows that no group idiosyncratic risks impact the tested transaction pricing.

Equation (2.1) can be equivalently presented as follows:

(2.2) $$y_{i,r,\tau} + \left(\pi_{r^*,\tau^*} - \pi_{r,\tau}\right) = \left(\pi_{r^*,\tau^*} + \pi\right) + \sigma \times \varepsilon_i$$

The equation (2.2) presents the interest rate modelling equation, which is consistent with the interest benchmarking algorithm:

1. Construct the sample of comparable yield rates $y_{i,r,\tau}$;
2. Estimate the parameters $\pi_{r,\tau}$ using Bloomberg or Reuters yield series;
3. Perform the rating / term premium adjustments: $y^*_{i,r^*,t^*} = y_{i,r,\tau} + \left(\pi_{r^*,\tau^*} - \pi_{r,\tau}\right)$;
4. The average of the adjusted y^*_{i,r^*,t^*} yield sample, $\pi_{r^*,\tau^*} + \pi$ includes tested transaction risk premium and term premium (through π_{r^*,τ^*} coefficient) as well as the sector / country premium or group idiosyncratic risk premium (through π coefficient);
5. The range constructed based on the y^*_{i,r^*,t^*} sample takes into consideration the σ^g volatility parameter.

Note that the above equation models the levels of the yield rates. Alternatively, the model can be presented in logarithms

(2.3) $$\ln y^g_{i,r,\tau} = \ln \pi^g + \ln \pi_{r,\tau} + \sigma^g \times \varepsilon_i$$

Similar to the equation in yield levels, the above equation can be equivalently represented as follows.

(2.4) $$\ln\left(y_{i,r,\tau} \times \frac{\pi_{r^*,\tau^*}}{\pi_{r,\tau}}\right) = \ln \pi + \ln \pi_{r^*,\tau^*} + \sigma \times \varepsilon_i$$

The IRB approach for the model specification is similar to the above approach with the exception that the yield adjustment is performed using the multiplication factor:

(2.5) $$y^*_{i,r^*,t^*} = y_{i,r,\tau} \times \frac{\pi_{r^*,\tau^*}}{\pi_{r,\tau}}$$

The equations presented in this section demonstrate how the IRB analysis described in **Error! Reference source not found.** and Section 5 can be supported by formal statistical analysis. Note that while the statistical analysis provides the support for the expected average (median) value, the volatility parameter is estimated only approximately and normally the range is estimated conservatively. Specifically, the following observations should be made with respect to the estimated ranges:

1. The range under the MYCA approach is estimated based on the volatility in time domain of the estimated parameters $\pi_{r,\tau}$. The variability over time in the estimated $\pi_{r,\tau}$ parameters is normally significantly lower then the variability in cross-sectional yield data. The narrow MYCA range should be interpreted as a very conservative estimate of the actual range;
2. The reported range is often selected as the common range under alternative approaches (e.g. a common range under the ICS and CNS approaches). A common range is selected to ensure robustness of the median value but produces a conservative estimate of the range variation.

Based on the discussion above, it should be noted that the statistical modelling is applied to produce a robust estimate for the average market interest rates and only a proxy estimate for the interest rate variability. In practice, a conservative estimate for the range of market interest rates is applied.

2.2 Nelson-Siedel model

2.3 Affine model

2.4 Yield projections and forward rates

The projected or forward interest rates are often used in the credit rating or debt capacity assessment of the borrower to estimate projected leverage metrics of the borrower. The modelling is applied for the floating rate debt transactions to project the movement in the future base rates and respective future interest expense.

The forward rates generally differ from the projected rates. Forward rates are estimated based on the rational expectation theory (discussed in Appendix B.1) and derived from no-arbitrage pricing principle. The projected yield rates are estimated based on the forecasts reported by 3d-party financial institutions. The default approach is to use the forward rates. The rationale is that the floating rates are hedged with float-to-fixed interest rate swaps.

2.4.1 Forecasting yield rate term structure

The yield rate term structure is forecasted as follows.

1. Forecast the yield rates for selected maturity terms: 3 months (short-term), 5 years (medium term), and 10-years (long-term). The forecasts are estimated based on the forecast benchmarking analysis by reviewing the 3d-party forecast reported by financial and research institutions.[1]
2. The yield rates for other maturities are estimated by interpolating the benchmarked 3-months, 5-year and 10-year forecasts. A simple linear interpolation is typically reasonably accurate. Alternatively, the interpolation that is consistent with the NSM modelling approach can be applied. The interpolation equations are described in Appendix B.3.2.

[1] The forecasts for the sovereign debt yield rates are reported by financial institutions as part of new IFRS 9 financial reporting requirements.

A direct benchmarking of the yield rates is available only for the sovereign yield rates. The short-term sovereign yield rates can be applied as the proxy for the base rate. The sovereign yield term structure can be applied to estimate the term premium. The risk premium can be assumed to be constant and equal to current risk premium.

2.4.2 Forward yield rates

Section 3 Dynamic Models of Interest Rates

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Section 4 Bloomberg Market Yield Curves

Market yield curve indices are calculated and adjusted by Bloomberg using a set of standardized consistent assumptions on the constituent debt's terms and conditions, such as semi-annual interest payments and the 30/360-day count basis for US$ denominated bond and note transactions. Bloomberg applies rate adjustment and standardization procedures to remove the effects of different extra features and options, which are typically present in many individual securities (e.g., callability, putability, convertibility, principal amortization, etc.). The yield curve indices are estimated for specific industry sectors, credit ratings, and maturity terms.

This section describes the yield curve indices available through Bloomberg database and the methodology applied by Bloomberg to estimate the yield curves.

4.1 Selected list of Bloomberg yield curves

Bloomberg reports US yield curves for the following broad industry sectors.

- ► Broad industrial sector;
- ► Broad financial sector;
- ► Broad banking sector.

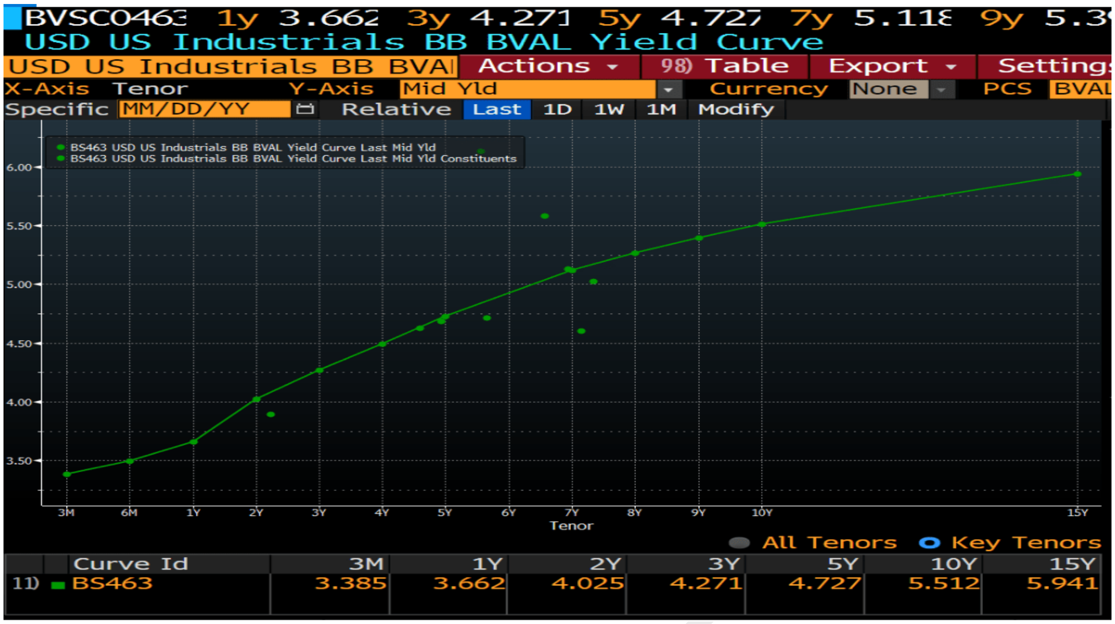
Bloomberg is using either BFV (Bloomberg fair value) or BVAL (Bloomberg value) functions to estimate the yield series. The two functions typically produce similar but not exactly the same yield estimates. BVAL function replaced the BFV function for the yield curves with sub-investment credit ratings (BFV curves were discontinued in 2013). The yield curves with investment grade credit ratings are reported for both BFV and BVAL functions.

The yield curves summarized below are estimated by Bloomberg using BVAL function. The yield curves are estimated for the range of credit ratings between B- to AA+. The range of maturity terms estimated by Bloomberg for each curve depends on the curve credit rating. Investment grade curves are estimated for the range of tenor terms between 3 month and 25 years. Sub-investment grade curves are estimated for the range of tenor terms between 3 month and 15 years. The curves for (BB) group and (B) group ratings are listed below.

- ► (BB) group: BVSC0463, BVSC0464, and BVSC0465 correspond respectively to BB, BB-, and BB+ industrial yield term structures estimated using Bloomberg BVAL function;
- ► (B) group: BVSC0463, BVSC0464, and BVSC0465 correspond respectively to B, B-, and B+ industrial yield term structures estimated using Bloomberg BVAL function.

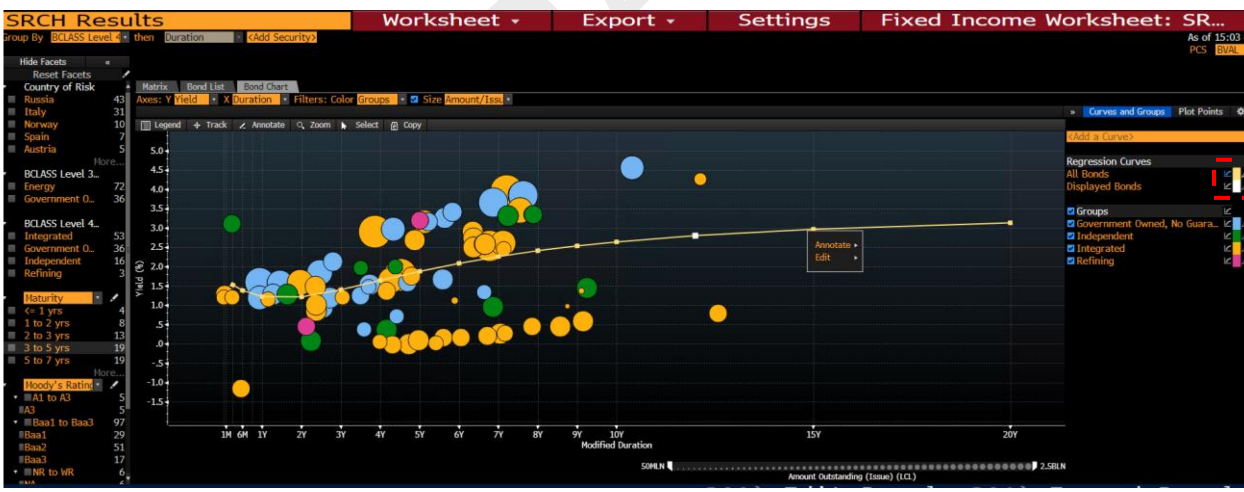
The corporate note samples used by Bloomberg for different yield curves can be obtained using Bloomberg MEMB function. Applying the MEMB function to the BVSC046 curve shows that the curve is estimated by Bloomberg based on eight constituents. The yield term structure and respective constituents' yields are illustrated in the Bloomberg print screen below.

The methods applied by Bloomberg to estimate the yield term structure based on the sample of constituents is described in Appendix B.



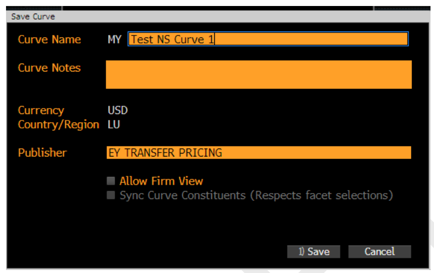
4.2 Estimation of Bloomberg custom yield curves

A sample of bonds identified through the search tool can be exported to Fixed Income Worksheet (FIW). The sheet includes the estimated bond curve and break-down of the bond sample by different categories.



The list of models applied to estimate the curve is shown below. The regression model is selected from the menu highlighted in the exhibit above. The list of models includes Nelson-Siegel (NS) as one of the options (discussed in this guide).

The created curve can be saved on Bloomberg.



The curve can be selected from the Curve Finder: ‘CRVF> Custom Curves’ menu option. Alternatively, the curve can be selected directly from the Custom Curve Builder (CRV) tool. The CRV tool allows to drop the list of securities directly from Excel.

The curve data can be saved to Excel by selecting ‘History Table’ option in the curve menu (highlighted above) and then copying the data at the bottom of the chart.

Alternatively, the curve can be presented as a table with ‘Values and Members’ option as shown below. The table data can be exported directly to Excel. This is a preferred option as it includes a large set of curve maturities.

MY TEST NS CURVE 1 | Actions | 98) Chart | Export | Settings | Graph Curves

X-Axis Duration | Y-Axis BVAL Yield | Currency None | PCS

Specific 04/24/20 | Relative Last 1D 1W 1M Modify | Curves & Relative Value

Values and Members | Values | Members | Constituents

		YCFC39KG BVAL Yield MY TEST NS CURVE 1 03/23/21		YCFC39KG BVAL Yield MY TEST NS CURVE 1 04/24/20		YCFC39KG BVAL Yield (Change) 03/23/21-04/24/20
	Tenor	Description	Yield	Description	Yield	Yield
11)	3M	MY Test NS Curve 1 3M	1.534	Same	4.145	-261.0
12)	6M	MY Test NS Curve 1 6M	1.390	Same	3.843	-245.2
13)	1Y	MY Test NS Curve 1 1Y	1.230	Same	3.529	-229.9
14)	2Y	MY Test NS Curve 1 2Y	1.225	Same	3.373	-214.8
15)	3Y	MY Test NS Curve 1 3Y	1.412	Same	3.374	-196.2
16)	4Y	MY Test NS Curve 1 4Y	1.651	Same	3.393	-174.2
17)	5Y	MY Test NS Curve 1 5Y	1.886	Same	3.409	-152.3
18)	[illegible]	Curve 1 6Y	2.094	Same	3.421	-132.7
19)	7Y	MY Test NS Curve 1 7Y	2.271	Same	3.429	-115.8
20)	8Y	MY Test NS Curve 1 8Y	2.420	Same	3.436	-101.6
21)	9Y	MY Test NS Curve 1 9Y	2.544	Same	3.441	-89.7
22)	10Y	MY Test NS Curve 1 10Y	2.648	Same	3.445	-79.7
23)	12Y	MY Test NS Curve 1 12Y	2.809	Same	3.451	-64.2
24)	15Y	MY Test NS Curve 1 15Y	2.974	Same	3.457	-48.3
25)	20Y	MY Test NS Curve 1 20Y	3.141	Same	3.463	-32.3

Click for Graph Curve Tenor (GCT) on all curves. Shortcut: GCT<GO>

The above view can also be used to show and export constituents. However, the constituents table does not show the yields of individual tickers.[2] The yields must be downloaded separately.

MY TEST NS CURVE 1 | Actions | 98) Chart | Export | Settings | Graph Curves

X-Axis Duration | Y-Axis BVAL Yield | Currency None | PCS

Specific 04/24/20 | Relative Last 1D 1W 1M Modify | Curves & Relative Value

Values and Members | Values | Members | Constituents

YCFC39KG MY TEST NS CURVE 1 BVAL Yield

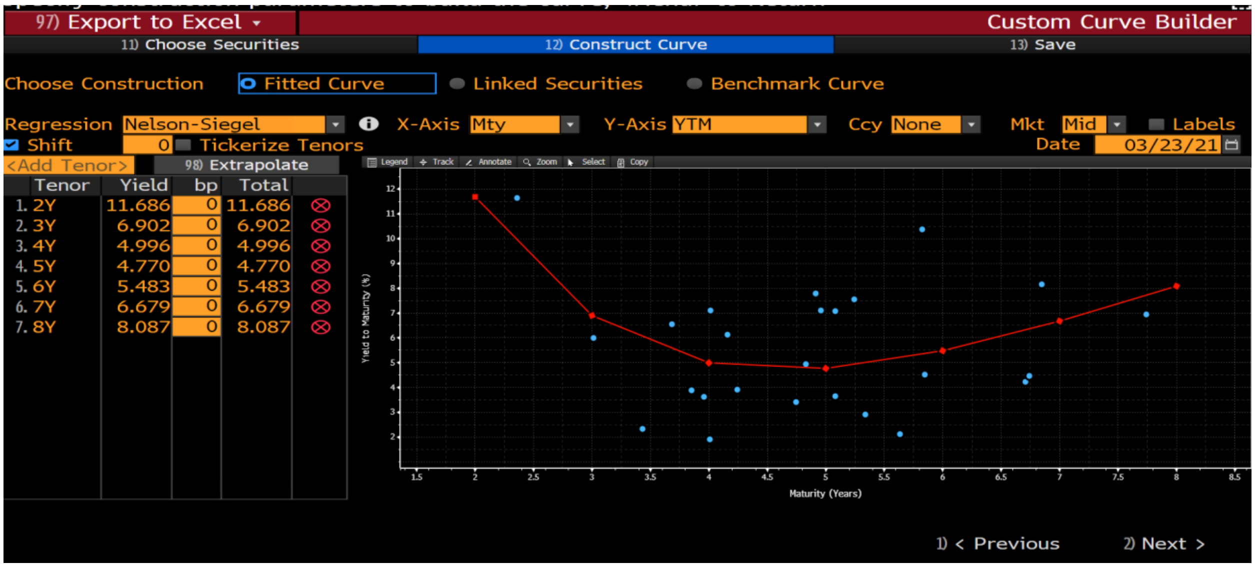
	CUSIP	Maturity↑	Sector	Rating	03/23/21 (108)	04/24/20 (84)
11)	AT364715	.011	Energy	BBB+	ENIIM Float 06/28/26	Same
12)	AT586774	.043	Energy	BBB+	ENIIM Float 07/09/27	Same
13)	AQ451821	.117	Energy	BBB+	ENIIM Float 06/15/26	Same
14)	00973RAD	.221	Energy	BBB-	AKERBP 4 3/4 06/15/24	Same
15)	AZ096707	.221	Energy	BBB-	AKERBP 4 3/4 06/15/24	Same
16)	UV853118	.463	Energy	BBB	OMVAV 5 1/4 PERP	Same
17)	368266AF	.929	Energy	BBB-	GAZPRU 6.51 03/07/22	Same
18)	EG235767	.929	Energy	BBB-	GAZPRU 6.51 03/07/22	Same
19)	77819RAA	.930	Energy	BBB-	ROSNRM 4.199 03/06/22	Same
20)	EJ465222	.930	Energy	BBB-	ROSNRM 4.199 03/06/22	Same
21)	549876AA	1.148	Energy	BBB	LUKOIL 6.656 06/07/22	Same
22)	EG518095	1.148	Energy	BBB	LUKOIL 6.656 06/07/22	Same
23)	368266AR	1.274	Energy	BBB-	GAZPRU 4.95 07/19/22	Same
24)	EJ281225	1.274	Energy	BBB-	GAZPRU 4.95 07/19/22	Same
25)	36192NAA	1.441	Energy	BBB-	SIBNEF 4 3/8 09/19/22	Same
26)	EJ359552	1.441	Energy	BBB-	SIBNEF 4 3/8 09/19/22	Same
27)	66989PAC	1.644	Energy	BBB	NVTKRM 4.422 12/13/22	Same

The constructed curve can be tracked at different dates and used as a custom yield curve for maturity term adjustments.

Alternative approach to custom yield curve estimation using CRV tool is illustrated in the exhibit below.[3] The exhibit above illustrates that the NS approach applied by Bloomberg potentially suffers from negative curvature problem. For a more detailed discussion of the NS estimation method see Appendix B.3.

[2] If the sample is accessed through the CRV tool, then the constituent yields can potentially be copied to Excel.

[3] Tool output is illustrated for a different sample.



97) Export to Excel
Custom Curve Builder
11) Choose Securities
12) Construct Curve
13) Save
Choose Construction
Fitted Curve
Linked Securities
Benchmark Curve
Regression Nelson-Siegel
X-Axis Mty
Y-Axis YTM
Ccy None
Mkt Mid
Labels
Shift 0
Tickerize Tenors
Date 03/23/21
<Add Tenor>
98) Extrapolate
Tenor Yield bp Total
1. 2Y 11.686 0 11.686
2. 3Y 6.902 0 6.902
3. 4Y 4.996 0 4.996
4. 5Y 4.770 0 4.770
5. 6Y 5.483 0 5.483
6. 7Y 6.679 0 6.679
7. 8Y 8.087 0 8.087
Yield to Maturity (%)
Maturity (Years)
1) < Previous
2) Next >

Section 5 Interest Rate Adjustments

Interest rate adjustments are performed after a sample of comparable loan transactions is identified. The step is important to ensure that the rates on comparable loans are adjusted for any material differences with the tested transaction.

The adjustments in this section are broken down into the following three groups.

1. Comparable-specific adjustments based on differences between the comparable notes and the tested transaction.
2. Tested transaction-specific adjustments which accounts for the options and provisions included in the tested transaction.

Adjustments in the first group are performed based on the statistical model selected to estimate option-free interest rates (see Section 2 and Section 3 for more details). Adjustments in the second group are performed using valuation tools designed specifically to estimate fair market value of options and provisions included in the loan agreement.

The section also discusses some standard tests that can be performed to validate the correctness of the performed adjustments.

5.1 Comparable-specific adjustments

This section discusses the adjustments which are performed to estimate option-free interest rates which are otherwise comparable to the interest rates on the tested transaction. The adjustments can be broadly broken down into the following two categories

(i) Convert the yield rates into option-free fixed rates, which are denominated in the same (US$) currency and have the same interest payment terms.

(ii) Apply curve-based adjustments to perform valuation date, credit rating and term premium adjustments.

The first step is performed to convert the rates on comparable loans into adjusted rates which are consistent with the yield rates applied to construct the yield curves. At the second stage, the adjusted (normalized) yields are applied (i) to model yield curves (or apply exogenously modelled yield curves) and (ii) to apply the yield curves to perform the valuation date, credit rating and term premium adjustments.

5.1.1 Interest rate swap adjustments

- Currency.
- Interest type.
- Interest payment frequency, day count basis, interest payment dates.

5.1.2 Issue price adjustments

Issue price different from par (100) value is normally observed in publicly traded 3rd – party notes but is rarely included in the 1st lien senior secured 3rd – party loans or intercompany loan/note. However, it may be observed in the 2nd-lien loan. The price discount is also referred to as 'original issue discount' (**OID**).

The issue price is adjusted in comparable sample using YIELD(…) Excel function. A high-level proxy for the OID adjustment can be assessed as follows. Suppose that a 5-year loan is issued at 99 discounted price. Then the $1 discount (=$100 par - $99 issue price) is the fixed cost to the borrower. The fixed cost is converted to the respective interest rate adjustment by dividing the $1 cost by the annuity adjustment factor (**AAF**). In the example, the adjustment will be slightly above 20bps = ($1 / 5 years, where 5 years is a high-level proxy for the AAF).

5.1.3 Option adjustments

5.1.4 Curve-based adjustments

The curve-based adjustments can be described by the following simple equation

$$\Delta y = \pi_{t^*,r^*,\tau^*} - \pi_{t,r,\tau} \tag{5.1}$$

where t and t^* are valuation dates, r and r^* are credit ratings, and τ and τ^* are maturity terms (estimated as of respective valuation dates). The adjustments are calculated based on respective yield series (estimated and reported by Bloomberg or Reuters and described by $\pi_{t,r,\tau}$ tables).

In practice, the adjustment can be broken down into the following components

$$\pi_{t^*,r^*,\tau^*} - \pi_{t,r,\tau} = \left(\pi_{t^*,r,\tau} - \pi_{t,r,\tau}\right) + \left(\pi_{t^*,r^*,\tau} - \pi_{t^*,r,\tau}\right) + \left(\pi_{t^*,r^*,\tau^*} - \pi_{t^*,r^*,\tau}\right) = \Delta y^{VD} + \Delta y^{CR} + \Delta y^{term} \tag{5.2}$$

where the first term is interpreted as the valuation date adjustment, the second term is interpreted as credit rating adjustment, and the last term is interpreted as maturity term adjustment.[4] Formally, the equation and write up for each individual adjustment are described as follows.

Valuation date adjustment

Equation

$$\Delta y^{VD} = \pi_{t^*,r,\tau} - \pi_{t,r,\tau} \tag{5.3}$$

Description

"The adjustments for the valuation date differences were made by estimating the differences in the market yield rates obtained as of the valuation date of the Loan and the valuation date of each related external

[4] Note that the sequence of adjustments does not affect the final result.

CUT. The market yield rates were estimated based on the Bloomberg's yield rate series with the credit rating and the maturity term matching the credit rating and the maturity term of each related external CUT."

Credit rating adjustment

Equation

(5.4) $$\Delta y^{CR} = \pi_{t^*,r^*,\tau} - \pi_{t^*,r,\tau}$$

Description

"The relevant interest rate adjustments were made to account for differences in transaction-specific credit ratings between the selected CNS CUTs and the Loan by calculating the spreads between (i) the r^*-rated US$ Industrial bond yield curve rates applicable to the Loan and (ii) the US$ Industrial bond yield curve rates with the credit rating of the CNS CUTs. The yield curves with the maturity terms matching the effective remaining tenors of the CNS CUTs and dated on t^* were used to perform the credit rating related adjustments".

Maturity term adjustment

Equation

(5.5) $$\Delta y^{term} = \pi_{t^*,r^*,\tau^*} - \pi_{t^*,r^*,\tau}$$

Description

"The maturity term adjustment for each CUT was performed by calculating spreads between (i) the τ^*-year r^*-rated US$ Industrial yield curve rate; and (ii) the r^*-rated US$ Industrial yield curve rates with the tenor term matching each CUT's effective remaining tenor. The yield rates dated on t^* were used to perform the maturity term adjustments".

5.2 Transaction-specific adjustments

5.2.1 Prepayment and pay-on-demand options.

5.2.2 Adjustment for amortization provision

5.2.3 Adjustment for interest deferral and PIK provision

A PIK, or payment in kind, is a type of high-risk loan or bond that allows borrowers to pay interest with additional debt, rather than cash. That makes it an expensive, high-risk financing instrument since the size of the debt may increase quickly, leaving lenders with big losses if the borrower is unable to pay back the loan. A PIK provision is effectively equivalent to capitalized interest deferral.

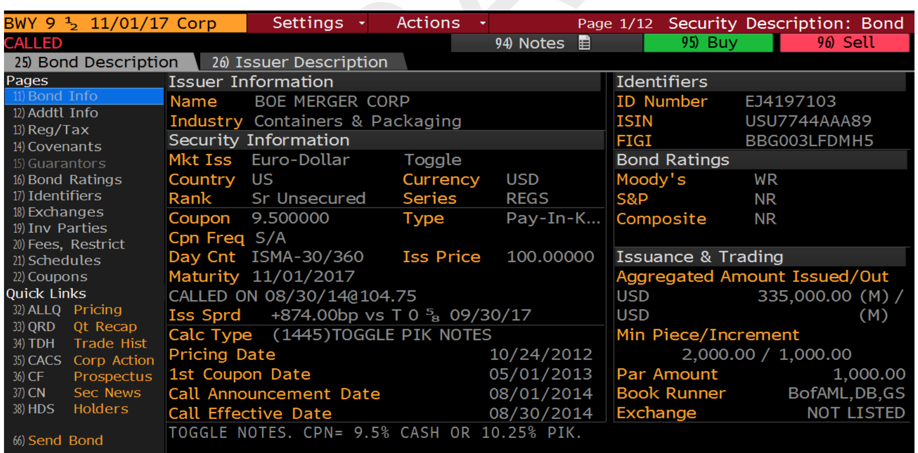
A standard approach to adjust for PIK provision is through credit rating adjustment. With PIK provision the tested debt transaction can be treated as a hybrid debt. The flexibility to defer interest payments in the PIK debt makes it similar to equity and results in a higher credit risk exposure for the lender. A PIK provision is typically accounted for by an additional one-notch downward adjustment of the tested transaction credit rating.

An alternative approach is to adjust for the PIK provision directly by estimating the spread between the PIK and cash interest. An example of the approach is illustrated below.

5.2.3.1 Toggle PIK notes

PIK toggles, also known as "pay if you want", are slightly less risky than PIKs, as borrowers pay interest in cash and may "toggle" to payment in kind at the discretion of the borrower ("pay if you want").[1] Sometimes, the borrower may also be able to PIK some portion of the interest (usually half) while paying the rest in cash; at times, only some of the interest may be paid in kind and the rest is cash-only. This also benefits borrowers, as they may opt for early payment of interest in cash, thereby minimizing the compounded payout at maturity. The documentation often provides that if the PIK feature is activated, the interest rate is increased by 25, 50, or 75 basis points.[5]

A sample search for toggle PIK notes indicated that 75bps is the most frequently observed spread between PIK and cash interest. Examples of toggle PIK notes with available prospectus describing the notes interest terms can be obtained through Bloomberg (e.g. the following Bloomberg tickers refer to toggle notes with available prospectus: EJ9912795, EJ6840270, EJ2268534, EI7780774, EJ4197103). A Bloomberg print screen with a toggle note description is illustrated below.



Description of a toggle note cash and PIK interest and related PIK over cash interest spread

The PIK premium estimation method can be summarized as follows:

- ► Search for toggle PIK notes using search criteria matching the terms of the tested loan and estimate the spread between the PIK and cash interest based on the identified sample of toggle notes;

[5] https://en.wikipedia.org/wiki/PIK_loan

- ► Perform a single extended search of toggle notes and apply the results of the search to each tested debt transaction with PIK component (current default recommendation is to apply **75bps** as a PIK / interest deferral premium).

5.2.3.2 Interest deferral

Interest deferral option is similar to a PIK provision but has several important differences.

1. Transfer pricing exposure.
2. Duration of interest deferral. Limited duration of interest deferral period;
3. Simple vs capitalized interest deferral. Deferred interest may be accrued but not capitalized.

Adjustment for interest deferral can be broken down into two separate components:

1. Adjustment for the additional risk exposure from delayed interest payments;
2. Adjustment for reduced actual interest payments.

The adjustment for the change in the loan cash flows, which results from the exercising the interest deferral option by the borrower, is performed using the NPV calculations. Not that the adjustment is generally non-zero only if the deferred interest is not capitalized. Otherwise, the adjustment is either zero or not material,

Adjustment for the increase in the risk exposure can be performed in several alternative ways. The approaches typically applied in transfer pricing analysis are summarized below,

1. Adjustment for deferral option risk premium through the adjustment of the tested loan transaction-specific credit rating;
2. Risk premium adjustment based on the search for toggle notes with PIK provision. Since the differed interest in toggle notes is capitalized, the difference between the PIK and cash interest in a toggle note represents the premium for the additional risk. The premium is paid only conditionally on exercising the PIK provision.

5.2.4 Country and sector premium adjustment

- ► Borrower's country.
- ► Borrower's industry sector.

5.2.5 Seniority / subordination

Adjustment for the tested loan subordinated ranking is typically performed by adjusting the loan transaction-specific credit rating. The methodologies applied for transaction-specific credit rating adjustments are described in the respective 'credit Rating Analysis' guide.

Alternatively, some intercompany loans are issued (or interpreted) directly as 2^{nd} – lien loans. Market data is generally consistent with significantly larger adjustment for the loan lower ranking compared to the premium assessed based on the credit rating adjustment. A separate search of the market loan financing deals, which include both 1st – lien and 2nd – lien loan issuances, may be recommended to assess the premium, which is consistent with the market. An example of such search performed for the North American issuers operating in the Oil & Gas industry is illustrated in Appendix **Error! Reference source not found.**.

5.2.6 Other considerations

In this section, we review the terms of a loan agreement, which generally do not have a direct impact on the interest rate but need to be examined as they may have an indirect impact though risk exposure or additional fees considerations.

- **Lender**. [AFR rates]
- **Borrower.** The creditworthiness of the borrower and the borrower's capital structure are the base to determine the credit rating of the tested transaction. Credit rating is one of the key factors which determines the applicable interest rates.

  [Capital structure and mezzanine debt]
- **Transaction format and business purpose**. The purpose of the transaction typically does not have a direct impact o the applicable interest rate. The loan format is selected based on the business purpose of the loan (see Section **Error! Reference source not found.** for a more detailed discussion). While the loan format does not affect the interest rates, it may require estimating additional applicable fees such as, for example, commitment or facility fee. The fees may be reported separately from the interest rate or may be blended as part of interest rate.
- **Principal amount**. The debt quantum has an indirect impact on the credit rating of the tested transaction and as a result on the estimated market interest rate. Principal amount can also be used as one of the screening criteria to ensure a better comparability of identified CUTs with the tested transaction. Otherwise no additional adjustment is performed for the size of the tested transaction.

  [Bloomberg adjustment for the transaction size]
- **Security / guarantee provision**. For the identified internal or external CUTs no adjustment is performed for a security or guarantee provision. Formally any positive benefit for the lender from secured or guaranteed CUT is reflected in the credit rating of the CUT and therefore no additional adjustment is required. In practice presence of a security provision in a CUTs may signal a higher credit risk of the CUT. The yields on secured CUTs may often be higher than the yields on the CUTs with the same credit rating and maturity term but no security provisions. To eliminate potential bias in the sample, out current standard practice is to remove secured CUTs from the final sample.

  If the tested transaction is secured or guaranteed, then the credit rating of the transaction should be adjusted accordingly. If the tested transaction is guaranteed by the parent company, then the parent can be viewed as the effective borrower in the transaction. The credit rating of the tested transaction is assessed then based on the parent credit rating. A higher credit rating of the parent compared to the credit rating of the borrower will results in a higher credit rating of the tested transaction and a lower market rate applicable to the transaction. Formally the reduction in the interest rate due to the guarantee provision should be compensated by the borrower to the parent through the intercompany guarantee fee (see the discussion of guarantee fees in the "Financial Guarantees" guide.[6]

[6] http://alexacomputing.com/files/other/fstp_guide/pdf/FSTP_04._Financial_Guarantees_v2.pdf.

5.3 Adjustment validation analysis

The list of IRB analysis validation tests is summarized below. The list is a suggested basic list which should be extended if necessary. The IRB analysis is often updated one or more times. In this case the consistency of the original and updated results should also be validated.

1. **Unadjusted** yields. If the IRB analysis is updated, then the movement in the unadjusted yield rates is the initial proxy for the overall change in the yield rates.
2. **Credit rating** adjustment. A basic test checks that adjustment to lower credit ratings is positive. Note that in the past Bloomberg would report periodically inconsistent yield series so that the credit rating adjustment test would fail if the adjustment were performed based on Bloomberg yields. Currently Bloomberg fixed the issue by setting the minimum spread of 2bps between two credit rating with a one-notch difference whenever the Bloomberg yield series valuation analysis would produce inconsistent yields for the two credit ratings.
3. **Maturity term** adjustment. Normally, the term structure of yield rates is upward sloping. The maturity term adjustment is tested by checking whether the adjustment to longer maturities is positive.
4. **Option** adjustment. The key parameters of the option are drift and volatility parameters. The change in the option values should be compared by comparing the changes in the volatility and drift parameters.
5. **Amortization** adjustment. Amortization option is effectively a term premium adjustment. The amortization template should include a proxy term premium adjustment which should be compared with the actual amortization adjustment (based for example on NPV analysis).

   If the IRB analysis (including amortization adjustment) are updated, then the change in the amortization adjustment should be compared against the movements in the term premium.
6. **Interest deferral** adjustment.
7. **Redraw option** adjustment (for loan facilities). The redraw option is typically performed based on the facility fee searches. The results typically change little over time and produce values in the 20 – 25 bps range. The values are annualized to estimate annual premium. Any material deviation from the numbers should be reviewed carefully.

Appendix A Fin 48 Reserve Calculation Analysis

FIN 48 (mostly codified at ASC 740-10) is an official interpretation of United States accounting rules that requires businesses to analyze and disclose **income tax risks**. It was effective in 2007 for publicly traded entities, and is now effective for all entities adhering to US GAAP.[7] FASB Interpretation No. 48, "Accounting for Uncertainty in Income Taxes" (FIN 48) requires companies to recognize, measure, present and disclose uncertain tax positions they take, or expect to take, in their tax returns. FIN 48 has significant practical and technical consequences because it applies to the most complex areas of tax. Effects go well beyond FIN 48's impact on amounts reported in financial statements because the accounting standard requires companies to disclose uncertain tax positions, including significant transfer pricing issues.[8]

In the context of transfer pricing analysis, the income tax risk refers to the potential risk of the tax authorities to challenge the intercompany allocations between the related companies under audit, including the allocations related to the intercompany interest expenses. In this section, Fin 48 analysis refers to the analysis of transfer pricing risk related to the tax authorities challenge of the interest rates applied on the intercompany loan transactions.

The interest rate challenge may be based on the following two rationales:

1. The tax authority disagrees with the consistency of the interest rate set intercompany loan with the market interest rates. The tax authority position is that a lower interest rate should have been applied since the loan issue date and respectively lower interest expenses should have been claimed over the loan life.
2. The tax authority does not disagree with the transfer pricing analysis of the intercompany interest rate. However, the loan has a prepayment option, which would provide an incentive to the borrower to refinance the intercompany loan in the even if the market interest rates decrease significantly over time. This scenario applies in the case when the interest rate on the loan was estimated and set in the economic environment of high financial volatility and respectively high market rates. Over time, the market interest rates revert back and decrease to their long run equilibrium.

Note that exposure in each specific period may be relatively small but it becomes material with each year of the intercompany loan staying outstanding and therefore being potentially subject to the interest rate reassessment. Suppose, for example, that a 10% fixed interest rate was set on a $100m intercompany loan. As the market interest rates decrease to 6%, the company faces a risk that the tax authorities will reduce the allowed interest rate from 10% to 6%. The allowed interest expense cost will be reduced from I_0 = 10% x $100m = $10m to I_1 = 6% x $100m = $6m. The company will be required to pay interest on the disallowed $4m = $10m - $6m interest expense. Assuming tax rate is 30%, the company will be required to pay additional C = 30% x $4m = $1.2m of taxes in each specific year. The cost will apply to each year when the loan stays outstanding and the interest rate is reassessed by the tax authority.

The objective of the Fin 48 analysis is to assess a potential transfer pricing risk from the interest rate reassessment and set respective reserves in the company balance sheet against potential losses. The analysis is normally performed by a tax team with the transfer pricing team assisting with the analysis of the market rates to which the existing rate can be potentially re reassessed. In the case of the intercompany loan with the prepayment option, the transfer pricing team performs a "**prepayment risk**" analysis and

[7] https://en.wikipedia.org/wiki/Fin_48.

[8] https://www.pwc.com/ca/en/services/tax/corporate-tax/transfer-pricing/fin-48-exposure.html.

respective transfer pricing exposure. With a stronger blend between the transfer pricing and tax analysis, the full Fin 48 reserve calculation analysis can be performed by the transfer pricing team.

The steps and the output of the Fin 48 reserve calculation analysis are illustrated below for the prepayment risk scenario. Suppose that an intercompany loan has a risk-free prepayment option and the market rates decreased significantly over time after the interest rate was set on the loan. Suppose also that the management of the company does not want to refinance the loan at lower rates and decides to set instead a reserve to manage the transfer pricing risk. The reserve calculation analysis is performed for each year as the loan stays outstanding and can potentially be audited by the tax authorities. The steps of the analysis are summarized as follows.

(i) Summarize the terms of the callable intercompany loans, which have a risk of interest rate reassessment

(ii) Assuming that the interest rates are monitored as of beginning of each new fiscal year, perform interest rate benchmarking analysis for the loans using the last business date of the previous fiscal year as the valuation date. Estimate the median and maximum values of the market interest rate ranges.

(iii) Assign the reassessment probabilities for the following scenarios: (i) no reassessment (the actual rate is consistent with the market rate ranges); (ii) the rate is reassessed to the maximum rate in the range; and (iii) the rate is reassessed to the median rate I the range. In the example below we assume that the probabilities for the three scenarios are assigned respectively to 25%, 30% and 45%.

(iv) Estimate the risk exposure taking into account timing of the loan being outstanding, expected potential reduction in the cost expense, and potential respective increase in the expected income taxes.

Assuming that the analysis is performed for a single loan with principal balance P and fixed interest rate i, the results of the analysis are summarized by the following exhibit.

Exhibit A.1 Output for the Fin 48 reserve calculation analysis

Outcome	Probability	Cumulative probability	Interest rate	Risk exposure	Cumulative exposure
No reassessment	25%	25%	i	$C_0 = 0$	$C_0 = 0$
Reassessed to maximum rate	30%	55%	i^{max}	$C_1 = \tau \times P \times (i - i^{max}) \times T$	C_1
Reassessed to median rate	45%	100%	i^{med}	$C_2 = \tau \times P \times (i - i^{med}) \times T$	$C_1 + C_2$
Expected loss				$\boldsymbol{C = 25\% \times C_0 + 30\% \times C_1 + 45\% \times C_2}$	

where τ is the income tax rate and T is the time the loan was outstanding during the covered fiscal year. The i^{med} and i^{max} are the median and the maximum rates of the interest rate range estimated under the interest rate benchmarking analysis.

Note that in most cases client is not willing to do the prepayment analysis as they view the interest rate monitoring analysis as excessive and related exposure to transfer pricing risk as low. The key parameters, which determine the magnitude of the risk exposure are (i) loan principal amount; (ii) number of years the loan was outstanding; (iii) the magnitude of reduction in the market interest rates (relative to the interest rate on the loan). It is recommended to perform a high-level assessment of the risk exposure to conclude whether the risk is material and whether the prepayment analysis should be recommended.

Appendix B Term Structure Estimation

Yield term structure is typically estimated whenever standard term structures from Bloomberg are not available. The estimation is performed for example for low credit ratings (CCC+ or lower) or for different industry sectors (which are narrower than broad industrial or financial sector). Term structure can also be estimated for a specific entity based on the debt transactions issued by the company.

B.1 Expectations and Liquidity Premium Theories

Under the Expectations Theory, bonds with different maturity terms are assumed to be perfect substitutes. Therefore, if a bond with a long-term maturity is replaced by a sequence of bonds with short-term maturities, a return on a long-term bond must be equal to a compounded return on a sequence of short-term bonds. If we denote the yield rate on short-term (three-month) bonds as $y_{t,1}$, the expected yield rate on short-term bonds in a future period $s > t$ as $y^e_{s,1}$e, and the yield rate on a bond with maturity τ as $y_{t,\tau}$, then the relationship between the actual and expected short-term yield rates and the yield rates with longer maturities can be described by the following equation.

(B.1) $$\left(1 + y_{t,\tau}\right)^\tau = \left(1 + y_{t,1}\right) \times \left(1 + y^e_{t+1,1}\right) \times \ldots \times \left(1 + y^e_{t+\tau-1,1}\right)$$

Based on the equation (B.1), we can derive the long-term yield rate $y_{t,\tau}$ with maturity term τ using a sequence of current and expected short-term yields. Therefore, under the Expectations Theory, the problem of a term structure forecast is reduced to the problem of properly forecasting future expected short-term yield rates. The complete term structure curve is then constructed using the sequences of short-term yield rates as per equation (B.1).

Under an alternative Liquidity Premium Theory, bonds with different maturities are not viewed as perfect substitutes and lenders generally require a premium for bonds with longer maturities to compensate for the inflation and interest rate risks. The term structure under the Liquidity Premium Theory is modelled as follows:

(B.2) $$y_{t,\tau} = y^e_{t,\tau} + \pi_{t,\tau}$$

where the term $y^e_{t,\tau}$ is estimated using equation (B.1) and the term $\pi_{t,\tau}$ represents a maturity-term premium component of the bond yield rates. Under this approach, the term structure forecast problem is broken down into two separate components: (i) forecasting the expected future short-term rates $y^e_{s,1}$ and calculating the $y^e_{t,\tau}$ components of the medium and long-term rates; and (ii) forecasting the term premiums $\pi_{t,\tau}$ for the medium and long-term rates

B.2 Factor models

A generic representation of an interest rate factor model can be described as the following latent factor model:

(B.3)
$$\begin{cases} F_t &= \mu + AF_{t-1} + \eta_t \\ y_{t,\tau} &= \Lambda_\tau F_t + \varepsilon_{t,\tau} \end{cases}$$

where t denotes periods of time, τ denotes maturity terms, and $y_{t,\tau}$ denotes the observed term structure of yield rates, which is determined by factors $F_t = \{f_{1.t}, \dots, f_{n,t}\}$.. The factors F_t are not observed directly (and therefore referred to as latent variables). The first equation describes a first-order vector auto-regression (**VAR**) model for the latent factors. The second equation describes a linear relationship between the latent factors and the yield rates with different maturity terms.

Under the Kalman filter modeling framework, factors F_t are interpreted as state variables (that are not observed directly), and variables $y_{t,\tau}$ are interpreted as observed measurements of the state variables. The first equation in the above system models the change in state variables as a VAR process, and the second equation models a linear relationship between the state variables and their measurements.

A large number of approaches to term structure modeling can be reduced to the above system of equations (B.3). Below, we describe some standard specific cases of the above modeling framework.

B.3 Nelson-Siegel yield term structure model

The Nelson-Siegel term structure model (**NSM**) is a three-factor model, in which the term structure is described directly by equation (B.3). The three factors are interpreted as follows: (i) $f_{1,t}$ is the term structure *level* factor; (ii) $f_{2,t}$ is the term structure *slope* factor; and (iii) $f_{3,t}$ is the term structure *curvature* factor. Matrix Λ_τ is calculated as follows: $\Lambda_{1,\tau} = 1$, $\Lambda_{2,\tau} = \frac{1-e^{-\alpha\tau}}{\alpha\tau}$, and $\Lambda_{3,\tau} = \frac{1-e^{-\alpha\tau}}{\alpha\tau} - e^{-\alpha\tau}$. Functions $\Lambda_{1,\tau}$, $\Lambda_{2,\tau}$, and $\Lambda_{3,\tau}$ are referred to as factor loadings. The linear relationship between the yield term structure and the unobserved factors can be described explicitly as follows:

(B.4)
$$y_{t,\tau} = f_{0,t} + f_{1,t}\left(\frac{1-e^{-\alpha\tau}}{\alpha\tau}\right) + f_{2,t}\left(\frac{1-e^{-\alpha\tau}}{\alpha\tau} - e^{-\alpha\tau}\right) + \varepsilon_{t,\tau}$$

The NSM is a very traditional approach to term structure modeling in the financial industry (see for example [1]). An example of the NSM approach implementation in the context of dynamic term structure modeling is provided in [2].

In matrix form, the equation (B.4) is presented as follows

(B.5)
$$\bar{y}_t = \Lambda_t \times \bar{f}_t + \bar{\varepsilon}_t$$

where

(B.6)
$$\Lambda_t = \begin{pmatrix} \dots & \dots & \dots \\ 1 & \lambda_{\tau,2} & \lambda_{\tau,3} \\ \dots & \dots & \dots \end{pmatrix}$$

The rows of the matrix Λ_t correspond to the sample of yields $y_{t,\tau}$ with maturity τ estimated at a specific date t. The factors have the following properties: $\lambda_{\tau,1} = 1$ is constant; $\lambda_{\tau,2} = \frac{1-e^{-\alpha\tau}}{\alpha\tau} \to 0$ as $\tau \to \propto$; and $\lambda_{\tau,3} = \frac{1-e^{-\alpha\tau}}{\alpha\tau} - e^{-\alpha\tau} \to 0$ as $\tau \to 0$ or $\tau \to \propto$. Therefore, the factors are interpreted as follows:

1. Factor $f_{0,t}$ is interpreted as a long-term yield;
2. Factor combination $f_{0,t} + f_{1,t}$ is interpreted as a short-term yield;
3. Factor $-f_{1,t}$ is interpreted as a long-term slope of the term structure.
4. Factor $f_{2,t}$ models the term structure curvature.

The following section discuss the NSM model estimation in the case of (i) static case (fixed date t) using linear regression model with inequality constraint, and (ii) dynamic case using semi-parametric panel data regression model with inequality constraints.

B.3.1 Term structure sensitivity to NSM factors

B.3.2 Linear mapping between yields and NSM factors

The NSM model can also be applied to interpolate the projected full term structure based on selected projected maturity terms (typically, 3-month, 5-years, and 10-years). The interpolation is performed in two steps:

1. Estimate the factors of the NSM model based on the three forecasted yields.
2. Estimate the full term structure based on the estimated factors of the NSM model.

In the case of government bond yields forecasting, the 3-month, 5-year, and 10-year yields can often be benchmarked using third-party forecasts. The equations below show how to forecast the yield rates with other maturities consistently with the NSM model.[9]

The NSM model factors are estimated by applying equation (B.4) to three specific maturity terms, 3-month, 5-year, and 10-years (denoted as $y_{t,3m}$, $y_{t,5y}$, and $y_{t,10y}$). The reverse linear relationship between the yield rates and the factors is described as follows:

(B.7)
$$\begin{pmatrix} f_{1,t} \\ f_{2,t} \\ f_{3,t} \end{pmatrix} = (\Lambda^T \Lambda)^{-1} \times \Lambda^T \times \begin{pmatrix} y_{t,3m} \\ y_{t,5y} \\ y_{t,10y} \end{pmatrix}$$

After we substitute equation (B.7) into equation (B.4), then we will generate a linear approximation of the term structure $y_{t,\tau}$ using the yield rates for three selected maturity terms of 3 months, 5 years and 10 years. The matrix $(\Lambda^T \Lambda)^{-1} \times \Lambda^T$ that maps the 3-month, 5-year, and 10-year yield rates into level, slope, and curvature factors (and vice versa) is described below.

1. Mapping between the 3-month, 5-year and 10-year yield rates and the level, slope, and curvature factors. The mappings are used to approximate the level, slope, and curvature factors using the 3-month, 5-year, and 10-year yield rates (and vice versa):

[9] Note that simple linear interpolation will also produce a reasonably accurate forecast. However, applying interpolation derived from the NSM model provides consistency if the NSM model is sued for the purpose of term structure modelling.

Mapping from the level, slope, and curvature to the full term structure							Mapping from the 3-month, 5-year and 10-year yield to the full term structure						
3-month		0.97	0.89	0.09		Level	Level		0.20	-1.47	2.23		3-month
5-years	=	0.98	0.25	0.26	x	Slope	Slope	=	1.09	0.71	-1.74	x	5-years
10-years		1.01	0.08	0.16		Curvature	Curvature		-1.83	8.80	-6.79		10-years

2. Mappings (i) from the 3-month, 5-year and 10-year yield to the full term structure (ii) from and the level, slope, and curvature factors to the full term structure. The mappings are applied to interpolate the full term structure using (i) the level, slope, and curvature factors (left panel); or (ii) 3-month, 5-year, and 10-year yield rates (right panel):

Mapping from the level, slope, and curvature to the full term structure							Mapping from the 3-month, 5-year and 10-year yield to the full term structure						
3-month		1.00	0.91	0.08			3-month		1.00	0.00	0.00		
6-month		1.00	0.84	0.14			6-month		0.86	0.39	-0.20		
1-year		1.00	0.71	0.23			1-year		0.60	0.93	-0.48		
2-years		1.00	0.53	0.29		Level	2-years		0.31	1.29	-0.56		3-month
3-years	=	1.00	0.41	0.29	x	Slope	3-years	=	0.15	1.29	-0.42	x	5-year
5-years		1.00	0.27	0.24		Curvature	5-years		0.00	1.00	0.00		10-year
7-years		1.00	0.19	0.19			7-years		-0.04	0.60	0.43		
10-years		1.00	0.14	0.14			10-years		0.00	0.00	1.00		
20-years		1.00	0.07	0.07			20-years		0.15	-0.79	1.65		
30-years		1.00	0.05	0.05			30-years		0.32	-1.41	2.03		

B.3.3 NSM estimation using regression analysis

This section discusses estimation of the NSM model in static case (fixed date t). We omit parameter t from the model equations. Three different cases are considered.

(i) Estimate the term structure for a given single credit rating assuming no constraints on the term structure;

(ii) Estimate the term structure for a given single credit rating assuming that the term structure is constrained by another given term structure;[10]

(iii) Estimate the term structure for a set of credit ratings so that the term structure for higher ratings are constrained from above by term structures with lower credit ratings.

The three cases are discussed below.

B.3.3.1 Unconstrained term structure for a single credit rating

A direct estimation of the NSM is performed based on equation (B.4). Estimation is performed as follows:

1. Perform the search for the yield rates y_τ;
2. Construct NSM factors $\lambda_{\tau,i}, i = 1,2,3$ for each yield observation y_τ;

[10] The approach is applied in the case of CCC+ rated term structure estimation, which is constrained by Bloomberg or Reuters B-rated term structure.

3. Estimate regression (B.4) using standard ordinary least square (**OLS**) estimation method;
4. Estimate yield term structure as $y_\tau^* = \sum_{i=1,2,3} \lambda_{\tau,i} \times f_i$;
5. Identify outliers y_τ which deviate significantly from y_τ^* and remove them from the sample. Repeat steps 2 – 4.

Example of unconstrained term structure estimation is provided in Appendix **Error! Reference source not found.**.

B.3.3.2 Constrained term structure for a single credit rating

When the term structure is estimated for example for a CCC+ (or lower) credit rating, certain constraints need to be taken account. Minimum required constraint is that estimated CCC+ term structure is higher than the B- term structure. Since NSM factors are linearly related to the yield rates, the constraint can be represented as

(B.8) $$f \times C \geq \bar{y}$$

where C is either a 10x3 or a 3x3 matrix described above and $\bar{y}$ is a yield constraint for the estimated term structure.

In certain cases, additional constraints may be added to ensure regular behavior of the estimated term structure and consistency of the IRB analysis. A standard additional constraint is positive slope of the estimated term structure. We assume a generic linear constraint in the NSM:

(B.9) $$f \times C \geq c$$

which includes both yield constraints, slope constraint or other linear constraints on the NSM factors. The constrained NSM is estimated using a constrained regression model:

$$\begin{cases} \min \frac{1}{2} \times (f\Lambda - y) \times (f\Lambda - y) \\ f \times C - c \geq 0 \end{cases}$$

Lagrangian of the constrained linear regression is described by the following equation

$$\mathcal{L} = -\frac{1}{2} \times (f\Lambda - y) \times (f\Lambda - y) + [f \times C - c] \times \vartheta$$

The solution of the constrained regression model is estimated as follows:

1. For a given $\vartheta \geq 0$, estimate vector

$$z = \Lambda \times \text{y} + C \times \vartheta$$

where initial Lagrange multiplier is selected as $\vartheta = 0$;

2. For iteration n, calculate OLS estimate for vector z using the following equation:

$$f^{(n)} = (\Lambda^T \times \Lambda)^{-1} \times z = f_0 + A \times \vartheta^{(n)}$$

where $f_0 = (\Lambda^T \times \Lambda)^{-1} \times (\Lambda \times \text{y})$ and $A = (\Lambda^T \times \Lambda)^{-1} \times C$;

3. Present the constraint as follows:

$$f^{(n)} \times C = f_0 \times C + C^T \times (\Lambda^T \times \Lambda)^{-1} \times C \times \vartheta^{(n)} \geq c$$

or equivalently

$$C^* \times \vartheta^{(n)} \geq c^*$$

where $C^* = C^T \times (\Lambda^T \times \Lambda)^{-1} \times C$ and $c^* = c - f_0 \times C$.

4. Estimate the constraint $\delta^{(n)} = f^{(n)} \times C - c$ and adjust ϑ as follows:

$$\ddot{\vartheta}_i = \vartheta_i - or\big(\vartheta_i > 0, and(\vartheta_i = 0, \delta_i < 0)\big) \times \gamma \times \delta_i$$

for a given parameter ϑ (where γ is capped by $\frac{\vartheta_i}{\delta_i}$). The equation states that Lagrange multiplier ϑ_i needs to be (i) increased whenever the constraint i does not hold, (ii) decreased whenever the constraints holds and $\vartheta_i > 0$; and (iii) not changed whenever the constraint holds and $\vartheta_i = 0$;

5. Repeat steps 1 – 3 until the following condition holds: $\vartheta \times \delta = 0$.

Example of unconstrained term structure estimation is provided in Appendix F.1.

B.3.3.3 Term structure for multiple credit ratings

In matrix form, the equation (B.4) is presented as follows

(B.10) $$y_i = \bar{f} \times \Lambda + \varepsilon_i$$

where the transposed of Λ is described as follows

(B.11) $$\Lambda^* = \begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 1 & \lambda^r_{\tau,2} & \lambda^r_{\tau,3} & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

and the estimated factors $\bar{f}$ are modelled as rating-specific factors appended to each other:

(B.12) $$\bar{f} = \left(f_0^{r_1}, f_1^{r_1}, f_2^{r_1}, \cdots, f_0^{r_n}, f_1^{r_n}, f_2^{r_n}\right)$$

where $r_1 \geq \cdots \geq r_n$ is the set of credit ratings for which the term structure is modelled. The constraints are modelled for a set of selected maturity terms: $\tau = 0.25, 5, 10$. The constraints are modelled using the following equations presented in the matrix form.

(B.13) $$\bar{f} \times C \geq 0$$

where matrix C for the 3-rating system is illustrated below.

(B.14)
$$C = \left(\begin{array}{ccc|ccc} -1 & -\lambda_{0.25,2}^{r_1} & -\lambda_{0.25,3}^{r_1} & & & \\ -1 & -\lambda_{5,2}^{r_1} & -\lambda_{5,3}^{r_1} & & 0 & \\ -1 & -\lambda_{10,2}^{r_1} & -\lambda_{10,3}^{r_1} & & & \\ \hline +1 & +\lambda_{0.25,2}^{r_2} & +\lambda_{0.25,3}^{r_2} & -1 & -\lambda_{0.25,2}^{r_2} & -\lambda_{0.25,3}^{r_2} \\ +1 & +\lambda_{5,2}^{r_2} & +\lambda_{5,3}^{r_2} & -1 & -\lambda_{5,2}^{r_2} & -\lambda_{5,3}^{r_2} \\ +1 & +\lambda_{10,2}^{r_2} & +\lambda_{10,3}^{r_2} & -1 & -\lambda_{10,2}^{r_2} & -\lambda_{10,3}^{r_2} \\ \hline & & & +1 & +\lambda_{0.25,2}^{r_3} & +\lambda_{0.25,3}^{r_3} \\ & 0 & & +1 & +\lambda_{5,2}^{r_3} & +\lambda_{5,3}^{r_3} \\ & & & +1 & +\lambda_{10,2}^{r_3} & +\lambda_{10,3}^{r_3} \end{array}\right)$$

Example of unconstrained term structure estimation is provided in Appendix F.2.

B.3.4 NSM estimation using panel data model with variable coefficients

B.4 Affine term structure models

Affine term structure models (**ATSM**), or no-arbitrage models, are a popular approach to model interest rates because the term structure derived under this approach is arbitrage-free. The affine structure of interest rates is modelled as follows. At the first step, short-term yield rates $y_{t,1}$ are described using n latent variables $f_{1.t}, \dots, f_{n,t}$ as follows:

(B.15)
$$y_{t,1} = \sum_{i=1,..,n} f_{i,t}$$

where each latent factor $f_{i,t}$ is described as follows[11]

(B.16)
$$\Delta f_{i,t} = k_i \times \left(\vartheta_i - f_{i,t}\right) + \sigma_i \sqrt{f_{i,t}} \eta_{i,t}$$

Yield rates with longer maturity terms are derived from short-term yield rates assuming that no arbitrage opportunities exist for bonds with longer maturity terms. An arbitrage-free price of a zero-coupon bond with maturity term τ, denoted as $P_{t,\tau}$, is described by the following equation

(B.17)
$$P_{t,\tau} = e^{\sum_{i=1,\dots,n}\left(A_{i,\tau} - B_{i,\tau} f_{i,t}\right)}$$

Respectively, yield rates on longer-term zero-coupon bonds are calculated as follows

[11] The ATSM are usually modelled assuming continuous time. This literature review covers a discrete-time approximation of the model.

$$y_{t,\tau} = -\frac{\ln P_{t,\tau}}{\tau} = -\frac{\sum_{i=1,\dots,n} A_{i,\tau}}{\tau} + \sum_{i=1,\dots,n} \frac{B_{i,\tau}}{\tau} \times f_{i,t} \qquad \text{(B.18)}$$

Equation (B.16) is a VAR model for the unobserved factors, and equation (B.18) describes a linear relationship between the unobserved factors and the yield term structure curve.

Equations (B.16) and (B.18) of the ATSM model can be represented as a special case of equation (B.3). Parameters A and Λ in the ATSM model specification are described as follows:

1. Matrix A is a diagonal matrix with $A_{i,i} = 1 - k_i$ diagonal elements;
2. Elements $\Lambda_{i,\tau}$ of matrix Λ are calculated as $\Lambda_{i,\tau} = \frac{B_{i,\tau}}{\tau}$, where $B_{i,\tau} = \frac{2\times(e^{-\gamma_i\tau}-1)}{(\gamma_i+k_i+\lambda_i)\times(e^{-\gamma_i\tau}-1)+2\gamma_i}$, $\gamma_i = \sqrt{(k_i+\lambda_i)^2+\sigma_i^2}$, and parameter λ_i is interpreted as the market price of risk.

Appendix C Estimation of Forward Term Structure

In some cases, for modelling purposes it is required to estimate a forward yield term structure, i.e. a term structure of yields which would be effective at a certain future period of time. The forward term structure can be estimated based on the expectation or liquidity premium term structure theory discussed in Appendix B. This section describes to alternative approaches to forward term structure estimation. An illustrative example of the forward term structure estimation is provided in Appendix G.

Appendix D Estimation of Risk-Free Rates

Risk-free rates are typically estimated for the purpose of discount factor calculations in various financial valuation models.

D.1 Bloomberg curves

Bloomberg curves that can potentially be used for risk free interest rate estimation.

1. US government bonds yield rate (C082 curve)
2. AAA rated financial (BVCVOI) or industrial (BVCVPO) yield rate curve
3. Libor swap curve (USSW curve)
4. Overnight swap curve (USSO curve)

Bloomberg snapshots with the curves' descriptions provided below.

C082 curve

BFV USD US Treasury Bonds/Notes 10 Year

*Effective May 01, 2007, US Treasury fair market curves 81, 82 and 83 were consolidated into curv 82. See ALLX C082<GO>. Fair marke value indices are derived from data points on Bloomberg's option-free Fair Market Curves. To vie the underlying curve and securities type, <CORP> BFVC 82. The yield at each maturity point represents the composite yield of securities around that maturity. Effective May 1, 2014, the Bloomberg Fair Value product was discontinued. Beginning on May 1, 2014 FMC Indices are derive from the BVAL Curves product. The indices are composite yields derived from BVAL-priced bonds. Additional information on the BVAL Curves product can be found at {BVLI <Go>}. To search for available curves and their indices, please run {CRVF <Go>}. FMC Indices with a last update prior to May 1, 2014 have been discontinued.

BVCVOI

BVCVOI10 BVLI Index | Security Description: Index

USD US Financials AAA BVAL Curve 10 Year

1) Notes

The yield curve is constructed daily with bonds that have BVAL prices at the market close.

Details	
Ticker	BVCVOI10 BVLI Index
Quote Type	Percent /Mid
Currency	USD
Country	
FIGI	BBG0081JDZX1
Source	BVAL
Price Frequency	Daily
End of Week	Friday
Start Date	02/24/2016

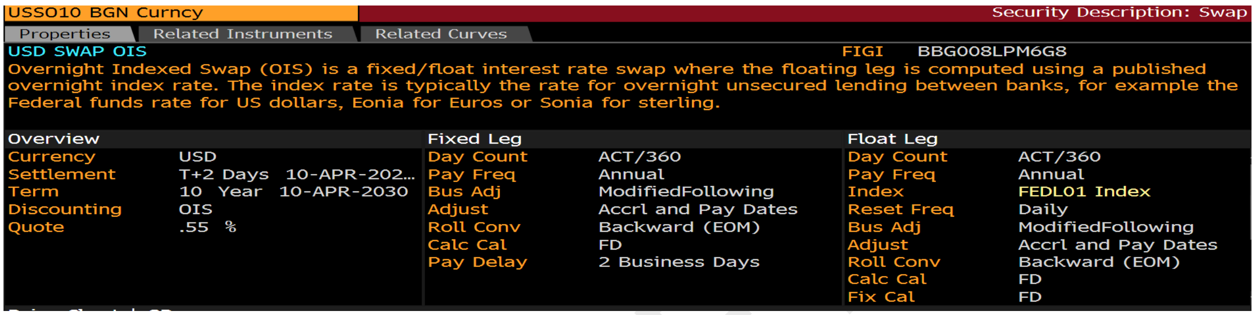
2) Line Chart | GP »

USSW curve


USD Semi Annual 30/360 (vs. 3M LIBOR) FIGI BBG00NTKL8L9
A vanilla interest rate swap is an agreement between two counterparties to exchange cashflows (fixed vs floating) in the same currency. This agreement is often used by counterparties to change their fixed cashflows to floating or vice versa. The payments are made during the life of the swap in the frequency that is pre-established by the counterparties.

Overview		Fixed Leg		Float Leg	
Currency	USD	Day Count	30I/360	Day Count	ACT/360
Settlement	T+2 Days 14-APR-202…	Pay Freq	SemiAnnual	Pay Freq	Quarterly
Term	10 Year 14-APR-2030	Bus Adj	ModifiedFollowing	Index	US0003M Index
Discounting	OIS	Adjust	Accrl and Pay Dates	Reset Freq	Quarterly
Quote	.8092 %	Roll Conv	Backward (EOM)	Bus Adj	ModifiedFollowing
		Calc Cal	FD, EN	Adjust	Accrl and Pay Dates
		Pay Delay	0 Business Days	Roll Conv	Backward (EOM)
				Calc Cal	FD, EN
				Fix Cal	EN

OSSO curve


USSO10 BGN Curncy Security Description: Swap
Properties Related Instruments Related Curves
USD SWAP OIS FIGI BBG008LPM6G8
Overnight Indexed Swap (OIS) is a fixed/float interest rate swap where the floating leg is computed using a published overnight index rate. The index rate is typically the rate for overnight unsecured lending between banks, for example the Federal funds rate for US dollars, Eonia for Euros or Sonia for sterling.

Overview		Fixed Leg		Float Leg	
Currency	USD	Day Count	ACT/360	Day Count	ACT/360
Settlement	T+2 Days 10-APR-202…	Pay Freq	Annual	Pay Freq	Annual
Term	10 Year 10-APR-2030	Bus Adj	ModifiedFollowing	Index	FEDL01 Index
Discounting	OIS	Adjust	Accrl and Pay Dates	Reset Freq	Daily
Quote	.55 %	Roll Conv	Backward (EOM)	Bus Adj	ModifiedFollowing
		Calc Cal	FD	Adjust	Accrl and Pay Dates
		Pay Delay	2 Business Days	Roll Conv	Backward (EOM)
				Calc Cal	FD
				Fix Cal	FD

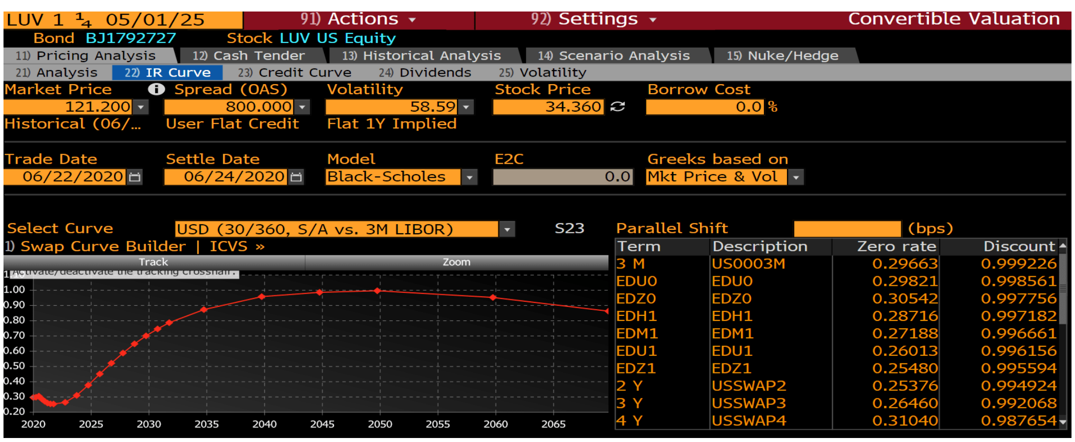
The difference between the overnight index swaps, Libor swap rates (USSW curve), AAA rated Financial yield series, and US government debt series (C082 curve).

D.2 Bloomberg risk-free rate estimation models

Bloomberg estimate curves applied for risk-free discount factor calculations in different tools such as for example interest rate swaps (SWPM) or valuation of convertible bonds (OVCV). The Bloomberg print screens with risk-free rate estimation within the valuation tools are illustrated below.

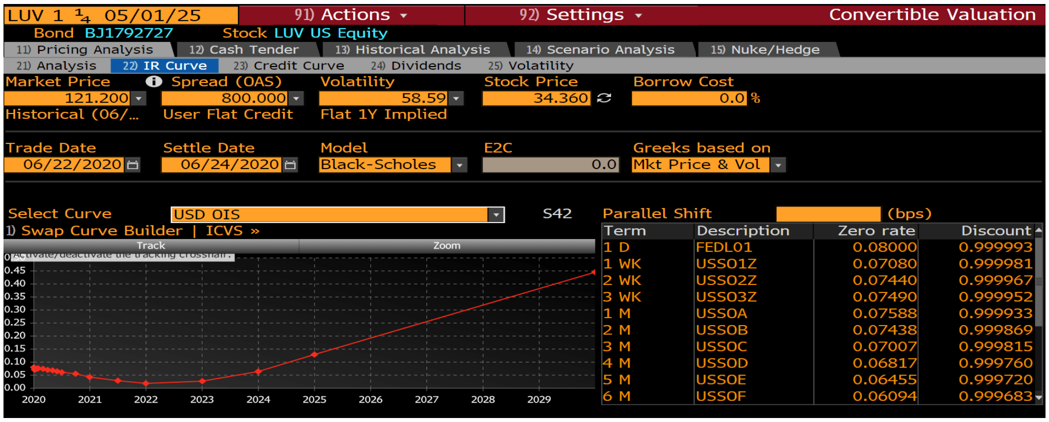
Risk-free rate estimation in OVCV tool

Risk-free rates are estimated either based on Libor swap curves



Term	Description	Zero rate	Discount
3 M	US0003M	0.29663	0.999226
EDU0	EDU0	0.29821	0.998561
EDZ0	EDZ0	0.30542	0.997756
EDH1	EDH1	0.28716	0.997182
EDM1	EDM1	0.27188	0.996661
EDU1	EDU1	0.26013	0.996156
EDZ1	EDZ1	0.25480	0.995594
2 Y	USSWAP2	0.25376	0.994924
3 Y	USSWAP3	0.26460	0.992068
4 Y	USSWAP4	0.31040	0.987654

or based on OIS curves



Risk-free rate estimation in SWPM tool

Appendix E AC.finance.IRB Tool

The section describes the tools developed as part of this guide. The objective of the tools is to implement the framework for interest rate benchmarking analysis. The framework removes the low-level coding and allows to create and manipulate high-level objects (such as a sample, a curve, sample yield adjustment, etc.) directly in Excel. As a result, the framework allows for a more efficient and clearer implementation of the interest benchmarking analysis.

A standard interest rate benchmarking analysis includes the following steps:

1. Create a tested transaction;
2. Create a sample of (comparable) transactions (CUTs) used for benchmarking the tested transaction;
3. If necessary, create a sample screening object and screen a sample of CUTs;

E.1 Inputs

The following inputs are used in a standard interest rate benchmarking analysis.

- **Sample fields**. Tested transaction and sample CUTs field data. The required fields are
  - BB_TICKER – used as the id of the transaction;
  - CRNCY – transaction currency;
  - VALUATION_DT – valuation date of the transaction;
  - MATURITY – transaction maturity date;
  - TENOR – parameter is set if fixed tenor is used for the CUT (parameter is used for example in the case when a generic yield curve with fixed tenor is used as a CUT);
  - CPN_TYP – coupon type (fixed or floating);
  - CPN_FREQ – coupon frequency;
  - DAY_CNT_DES – day count basis used for the CUT interest calculations;
  - RTG – consensus credit rating of the CUT (used as a default rating if ratings are not set separately;

  The tab with the parameters is illustrated below.

Exhibit E.1 Selected sample parameters used in IRB analysis

#	Transaction-specific Bloomberg code	Currency	Maturity Date	Effective Remaining Maturity Term (in years)	Interest Type	Interest (%)	Interest Payment Frequency	Day Count Basis	Consensus Credit Rating
	BB_TICKER	CRNCY	MATURITY	TENOR	CPN_TYP	CPN	CPN_FREQ	DAY_CNT_DES	RTG
			date	x.xx		x.xx	x		
CNS sample, which interest rates are benchmarked									
Covered Transaction	CT-1	CAD		3.0			1	ACT/365	BBB
CNS sample used to benchmark the interest rates									
1	EJ444777 Corp	USD	1-Feb-20	1.1	FIXED	2.625	2	30/360	BBB
2	EJ921121 Corp	USD	15-Nov-23	4.9	FIXED	4.25	2	30/360	BBB
3	QJ902839 Corp	USD	1-Dec-22	3.9	FIXED	4	2	30/360	BBB
4	EJ290406 Corp	USD	1-Aug-22	3.6	FIXED	3.15	2	30/360	BBB
5	EJ601223 Corp	USD	15-Apr-23	4.3	FIXED	3.75	2	30/360	BBB

- **Sample ratings**. Tested transaction and CUTs credit ratings. Sample ratings are used as inputs only if the dynamic IRB analysis is performed for multiple periods. Otherwise the static ratings from the tab with the sample fields are used.

  The (dynamic) ratings are set for each transaction as a sequence of date and consensus rating pairs, where each date is the effective date of the rating change or update published by Bloomberg. The tab with ratings parameters is illustrated below. (Bloomberg consensus rating notation is used to set input ratings).

Covered Transaction		*CNS Sample*									
		1		2		3		4		5	
CT-1		EJ444777 Corp		EJ921121 Corp		QJ902839 Corp		EJ290406 Corp		EJ601223 Corp	
20-Dec-18	BBB	20-Dec-18	BBB	20-Dec-18	BBB	20-Dec-18	BBB	20-Dec-18	BBB	20-Dec-18	BBB

- **Sample yields**. CUTs yield rates. Sample yields are used as inputs only if the dynamic IRB analysis is performed for multiple periods. Otherwise the static yields from the tab with the sample fields are used.

  The yields are set for each CUT as a sequence of date and yield pairs. The tab with yields parameters is illustrated below.

EJ444777 Corp		EJ921121 Corp		QJ902839 Corp		EJ290406 Corp		EJ601223 Corp	
8-Jun-18	3.32	4-Jun-18	4.12	4-Jun-18	3.89	4-Jun-18	3.70	4-Jun-18	3.93
11-Jun-18	3.34	5-Jun-18	4.10	5-Jun-18	3.86	5-Jun-18	3.70	5-Jun-18	3.72
12-Jun-18	3.36	6-Jun-18	4.13	6-Jun-18	3.91	6-Jun-18	3.75	6-Jun-18	4.15
13-Jun-18	3.39	7-Jun-18	4.09	7-Jun-18	3.88	7-Jun-18	3.70	7-Jun-18	4.06
14-Jun-18	3.38	8-Jun-18	4.11	8-Jun-18	3.88	8-Jun-18	3.73	8-Jun-18	4.13
15-Jun-18	3.37	11-Jun-18	4.12	11-Jun-18	3.88	11-Jun-18	3.74	12-Jun-18	4.16
18-Jun-18	3.38	12-Jun-18	4.14	12-Jun-18	3.89	12-Jun-18	3.74	14-Jun-18	4.17
19-Jun-18	3.38	13-Jun-18	4.17	13-Jun-18	3.91	13-Jun-18	3.76	21-Jun-18	3.98
20-Jun-18	3.41	14-Jun-18	4.14	14-Jun-18	3.90	14-Jun-18	3.71	25-Jun-18	3.86
21-Jun-18	3.37	15-Jun-18	4.12	15-Jun-18	3.90	15-Jun-18	3.72	26-Jun-18	3.94

► **Curves**. The following curves are used as inputs in the tool:

► **Yield curves.** The yield curves are used for rating and maturity adjustments of the CUTs' yields. The yield curves are set for the credit ratings and maturity terms consistently with the CUTs and tested transaction ratings and maturity terms. The currency and industry sector of the yield curves is selected consistently with the currency and industry sector of the CUTs. The inputs for a yield curve is illustrated below.

USD Industrial BBB- Yield Curve (IGUUABxx)

	3M		6M		1Y		2Y		3Y	
Date	Mid Price	Date	Mid Price	Date	Mid Price	Date	Mid Price	Date	Mid Price	Date
20-Dec-18	3.69	20-Dec-18	3.75	20-Dec-18	3.85	20-Dec-18	4.08	20-Dec-18	4.22	20-Dec-18
19-Dec-18	3.67	19-Dec-18	3.73	19-Dec-18	3.83	19-Dec-18	4.05	19-Dec-18	4.19	19-Dec-18
18-Dec-18	3.66	18-Dec-18	3.73	18-Dec-18	3.84	18-Dec-18	4.06	18-Dec-18	4.20	18-Dec-18
17-Dec-18	3.59	17-Dec-18	3.66	17-Dec-18	3.79	17-Dec-18	4.05	17-Dec-18	4.21	17-Dec-18
14-Dec-18	3.57	14-Dec-18	3.65	14-Dec-18	3.77	14-Dec-18	4.04	14-Dec-18	4.21	14-Dec-18

► **FX spot and forward data**. The FX data is used for cross-currency swap calculations. The FX spot rates and FX forward curves are set for the currencies consistently with the currencies of the CUTs and tested transaction. The tab with the inputs for a FX spot series and FX forward curve is illustrated below.

CAD		CAD12M		CAD2Y		CAD3Y		CAD4Y		CAD5Y
Date	Last Price	Date	Last Price	Date	Last Price	Date	Last Price	Date	Last Price	Date
21-Dec-18	1.36	21-Dec-18	1.35	21-Dec-18	1.34	21-Dec-18	1.34	21-Dec-18	1.33	21-Dec-18
20-Dec-18	1.35	20-Dec-18	1.34	20-Dec-18	1.33	20-Dec-18	1.33	20-Dec-18	1.32	20-Dec-18
19-Dec-18	1.35	19-Dec-18	1.34	19-Dec-18	1.33	19-Dec-18	1.33	19-Dec-18	1.32	19-Dec-18
18-Dec-18	1.35	18-Dec-18	1.34	18-Dec-18	1.33	18-Dec-18	1.32	18-Dec-18	1.32	18-Dec-18

► **Swap data**. The swap data is used to estimate discount rates. The discount rates are applied to perform swap calculations for interest payment differences (differences in interest rate frequency and day count basis). The currency of the swap data must match the currency of the yield curves. The tab with the swap data inputs is illustrated below.

USSW1		USSW2		USSW3		USSW4		USSW5
Date	Last Price	Date	Last Price	Date	Last Price	Date	Last Price	Date
21-Dec-18	2.84	21-Dec-18	2.80	21-Dec-18	2.75	21-Dec-18	2.73	21-Dec-18
20-Dec-18	2.86	20-Dec-18	2.83	20-Dec-18	2.78	20-Dec-18	2.76	20-Dec-18
19-Dec-18	2.81	19-Dec-18	2.77	19-Dec-18	2.72	19-Dec-18	2.70	19-Dec-18
18-Dec-18	2.80	18-Dec-18	2.78	18-Dec-18	2.74	18-Dec-18	2.73	18-Dec-18
17-Dec-18	2.84	17-Dec-18	2.84	17-Dec-18	2.81	17-Dec-18	2.80	17-Dec-18

The curve values for the maturity terms different from the maturity curves of the standard curves are estimated as follows.

- ► If a standard curve with higher and lower maturity term is available, then the curve value is interpolated using the values of the standard curves;
- ► If the maturity term is higher (lower) than the highest (lowest) maturity term of available standard curves, then the value of the curve is set equal to the value of the standard curve with highest (lowest) maturity;

If the yield curve with a credit rating matching a credit rating of a CUT is not set (the adjustment for the CUT will return a NaN value). Similarly, NaN values will be returned if the FX forward curve is not set for a currency matching the currency of a CUT.

E.2 Objects

The tool uses the following objects to perform interest benchmarking analysis.

- ► **Objects modelling transactions**
  - ► **Sample (static)**. A sample is the key object used in the interest benchmarking analysis. A sample object is created to model an initial sample of CUTs. Each transformation of the CUTs' yields produces a new sample. Effectively interest benchmarking analysis can be viewed as a sequence of transformed samples.

    A sample object is typically created in two stages. At stage one a static sample object with static sample fields is created. At stage two, dynamic data, such as ratings and yields, is added to the sample object. A static sample object uses the following input fields.

    fields ids;

    field names;

    field formats;

    field values;

    indicator of whether the field is numeric, a date, or other; and

    an indicator whether the field is included in the output table or not;
  - ► **Sample (dynamic)**. To create a dynamic sample object, the yields and ratings series are added to the sample transactions. If ratings are not included in the inputs, the static rating set in static sample object is also assumed to be applicable in each date. Maturity terms are calculated in the dynamic sample object based on the value of the static maturity term field. If the maturity term is not set, then the maturity term is set constant and equal to the value of the static tenor field.

    Formally, a dynamic sample object uses the following inputs:

    A static sample object;

    A set of tickers [redundant – remove in future];

    A range with CUTs historical yields;

    A range with CUTs historical ratings.
  - ► **Note**. A note object is typically not created or used directly. A note object is an element of a sample object.
- ► **Objects modelling inputs**
  - ► **Curve**. A standard object to model inputs is a curve object. The curve object model yield curves, FX forward curves, swap curves, etc. Effectively a curve can be viewed as a range of the

following triples: (date, tenor, value), which are set for a sequence of dates and sequence of tenor terms. A curve object uses the following inputs:

Curve id;

Curve type (*curve-yield*, *curve-swap*, or *curve-fx*);

Sequence of curve dates;

Sequence of curve tenor terms;

Range of curve data;

Curve parameters.

- **Series**. A series object is implemented to add modelling flexibility. For example, series objects are used for certain custom adjustments to the sample yields. A series object is created using the following inputs:

  Series id;

  Series currency;

  Sequence of series dates;

  Sequence of series values.

- **Calculator objects**. The actual interest benchmarking analysis is performed by calculator objects. The following calculators are used in standard analysis.
  - MYCA calculator.
  - CNS calculator;
  - FX swap calculator.
- **Output objects**.
  - **Samples**. A calculator object performs a sequence of sample transformations. The output sample objects are retrieved from the calculator object based on respective key that corresponds to a specific sample transformation. For example, the keys (*cns-sample-initial-swap, cns-sample-rating-adjusted, cns-sample-rating-adjusted, cns-sample-final-swap, cns-sample-fully-adjusted*) corresponds to output samples after respectively initial swap adjustment, rating adjustment, maturity term adjustment, final swap adjustment, and fully-adjusted sample.
  - **Static IRB output**. This is standard output of the IRB analysis, which shows the adjustment step and full-adjusted yield samples for a given valuation date. Output format is illustrated in the example in Section D.1.4.1.
  - **IRB ranges**. This is standard output of the IRB analysis, which shows the estimated range of the fully-adjusted yield rates and the change of the range over time. Output format is illustrated in the example in Section D.1.4.1.
  - **Sample yield series**. The output is viewed the yields at an intermediary adjustment step (for example USD yields adjusted for rating and maturity term but prior to the final swap adjustments).
  - **Sample adjustments**. An array of all performed adjustments is returned for a CUT with a given id. The output format is used to analyze the factors that affect the move in the interest rates (as illustrated in the example in Section D.1.4.2).

E.3 Implementation

The IRB tool implementation steps are summarized as follows.

- ► Construct input objects (yield curves, swap curves, discount curves, and the sample object).
- ► Construct IRB calculator object.
- ► Perform a sequence of sample yield adjustments. Each adjustment operation is implemented by an adjustment-specific class. The class creates a new sample with the adjusted yields. The adjustments are performed in the following order: (i) initial swap adjustment; (ii) valuation date adjustment; (iii) rating adjustment; (iv) tenor adjustment; (v) prepayment option adjustment; (vi) other adjustments; and (vii) final swap adjustment.
- ► Construct output objects.

E.4 Functions

This section describes steps of interest benchmarking analysis as a sequence of executed functions.

1. Create a static sample object: `AC.finance.irb.sample.fields(…);`
2. Create a dynamic sample object: `AC.finance.irb.sample.yields(…);`
3. Create input curve objects: `AC.finance.irb.curve(…);`
4. Create a calculator object.
   - ► MYCA calculator: `AC.finance.irb.myca.benchmark(…);`
   - ► CNS calculator: `AC.finance.irb.cns.benchmark(…);`
5. Retrieve calculator output.
   - ► Get a sample, yield data, or static output data: `AC.finance.irb.cns.get(…);`
   - ► Get a sample yield ranges: `AC.finance.irb.cns.range(…);`

E.5 Example A: standard interest rate analysis

Objective: the objective of the template is to produce standard output produced in a transfer pricing IRB analysis and integrate the output with the respective report.

An IRB analysis in a standard project is produced for a specific date and a specific sample that is estimated separately from the IRB tool. Standard output includes (i) sample yields and yield adjustments estimated at a given valuation date and (ii) ranges of yield rates. The output tables are illustrated below for the tested transaction denominated in C$ currency with BBB credit rating and 3-year maturity term.

Output A: Sample yields and yield adjustments.

The output is produced using `AC.finance.irb.cns.get(…)` function with the following inputs: (i) CNS calculator, key="*cns-output-standard*", and mapping: key=*'valuation-date*' => value=valuation date of the IRB analysis.

Exhibit E.2 Sample yields and yield adjustments

Issue Name	Currency	Rating	Tenor	Initial Yield	Initial Swap Adjustment	Rating Adjustment	Tenor Adjustment	Final Swap Adjustment	Fully Adjusted Yields
HCP INC	USD	BBB	1.12	3.46	0.00	0.00	0.42	-0.58	3.30
HCP INC	USD	BBB	4.90	4.26	0.00	0.00	-0.23	-0.58	3.45
HCP INC	USD	BBB	3.95	4.17	0.00	0.00	-0.12	-0.58	3.48
HCP INC	USD	BBB	3.61	3.95	0.00	0.00	-0.08	-0.58	3.30
HEALTHCARE REALTY	USD	BBB	4.32	4.26	0.00	0.00	-0.16	-0.58	3.52
HEALTHCARE TRUST	USD	BBB	4.32	4.08	0.00	0.00	-0.16	-0.58	3.34
HEALTHCARE TRUST	USD	BBB	2.57	3.78	0.00	0.00	0.07	-0.58	3.27
OMEGA HLTHCARE	USD	BBB-	4.61	4.48	0.00	-0.32	-0.20	-0.58	3.38
SENIOR HOUSING	USD	BBB-	1.32	4.98	0.00	-0.39	0.36	-0.58	4.37
SENIOR HOUSING	USD	BBB-	0.36	5.02	0.00	-0.44	0.62	-0.58	4.61
SENIOR HOUSING	USD	BBB-	2.99	4.91	0.00	-0.33	0.00	-0.58	4.01
VENTAS CANADA FIN	CAD	BBB+	0.78	2.77	0.79	0.14	0.50	-0.58	3.61
VENTAS REALTY LP	USD	BBB+	4.48	3.91	0.00	0.17	-0.18	-0.58	3.32

The exhibit presents only most typical adjustments of the sample yields. Other adjustments that are performed by the tool are (i) valuation date adjustment; (ii) other manual adjustments which may include country premium adjustment, adjustment for facility fee, and other. The tool displays only the columns with non-zero adjustments.

Output B: market yields and yield adjustments

The ranges of market yields and yield adjustments produced by the tool are illustrated in the exhibit below.

E.6 Example B: dynamic interest rate analysis

Objective: In the dynamic analysis, the changes in the ticker-specific adjusted rates and market interest rate ranges over time are estimated.

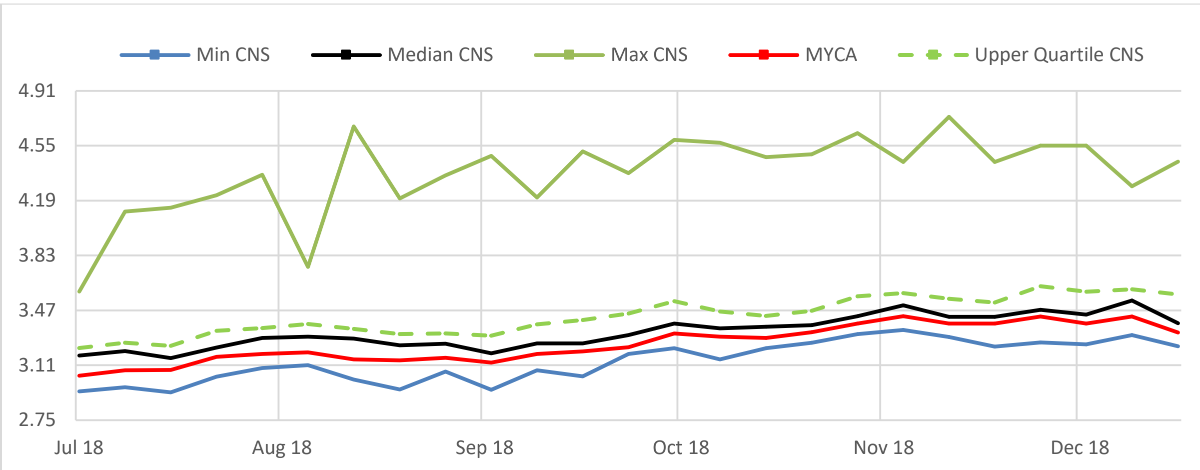
In addition to standard inputs (the curves and the sample), the tool requires to specify as an input the changes in credit ratings over time for each ticker in the sample.

The template for dynamic interest rates analysis is an extension of the standard IRB template. The template produces the standard output for each specific period of time but in addition produces the graphs and tables with the movements of interest rates over time.

Output A: Ranges of yield rates.

The output is produced using `AC.finance.irb.cns.range(…)` function with the following inputs: (i) sample with fully-adjusted yields, (ii) sample size (integer value, which specifies the minimum size of the sample used for range estimation, and (iii) frequency equal to one of the following values: *daily-frequency, weekly-frequency, monthly-frequency, quarterly frequency*, and *annual frequency*. Based on the frequency parameter, the yields are averaged over specific periods and ranges are estimated based on averaged yields.

Exhibit E.3 Dynamic ranges of fully adjusted yields



The graph below illustrates that the results of the MYCA analysis are consistent with the results under the CNS analysis. The graph shows also presence of outlier in the CNS sample which should be reviewed on individual basis. Consistency between MYCA and CNS results shows that there is no persistent premium in this specific sector.

E.7 Example C: break-down of interest rate changes

Objective: the purpose of the template is to assess the impact of different factors on the movement in the interest rates. The tool is typically used in the following cases:

- **Updated IRB analysis**. After the IRB analysis is updated, an additional assessment may be performed to estimate the factors that affect the change in the interest rates.
- **Internal CUT analysis**. The analysis is typically performed for a loan transaction. The interest rate on the internal CUT is adjusted to match the terms of the Covered Transaction.

The adjustments show then the impact of different factors on the change in the interest rates. Specifically, the impact of the following factors is estimated and presented in the template:

1. Valuation date adjustment (estimated through the change in the CUT yield over time);
2. Change in term premium (estimated through maturity term adjustment);
3. Change in risk premium (estimated through credit rating adjustment);
4. Change in cross-currency swap (estimated through the final swap adjustment);

The examples use the following objects:

- Tested transaction, which specifies the interest rate terms estimated under the IRB analysis;
- A CUT that models the movement in the market interest rates. The CUT could be a Bloomberg yield curve, an estimated yield series, or a specific corporate note yield series;

► Other standard input curves such as yield curves, FX forward curves, and swap curves.

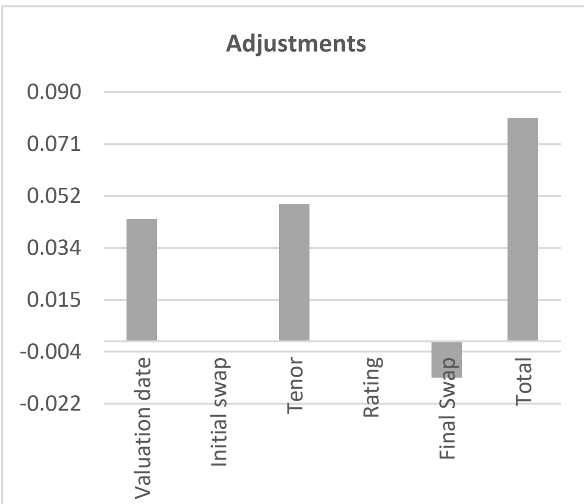
The change in each of the factor is presented using stacked column Excel diagram.

E.7.1 Example: updated IRB analysis

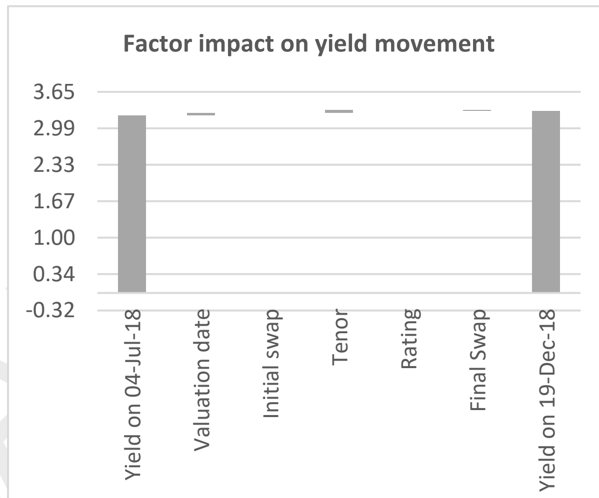
An example of the analysis and the output generated by the template is presented in the diagram below. In the example, a change in the 3-year CAD rate is estimated over time and the impact of each factor described above is estimated and presented. For illustrative purposes, we use the first CUT in the above sample (described in the previous example) as a market CUT.

Exhibit E.4 Yield adjustment analysis

Adjustment size



Adjusted yields



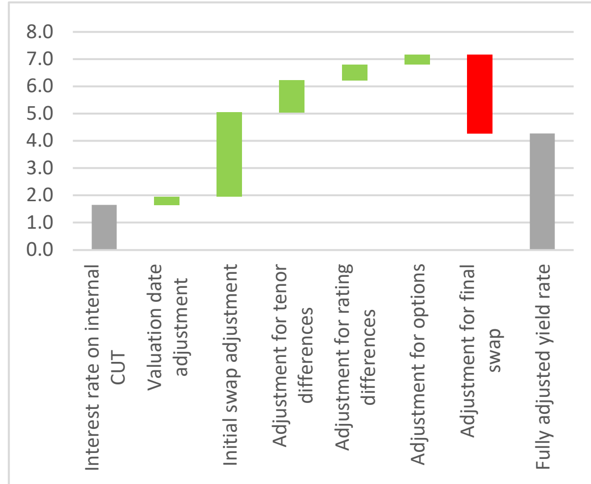
E.7.2 Example: Internal CUT analysis

In the internal CUT analysis, the interest rate on the third-party loan issued by the borrower or borrower's parent group is adjusted to estimate the interest rate on the intercompany loan. A summary of adjustments is illustrated in the exhibit below.

Exhibit E.5 Summary of interest rate adjustments for an internal CUT

Summary of interest rate adjustments

Interest rate on internal CUT	**1.65**	
Adjustments		
Valuation date	0.30	*30-Jul-18 to 14-Feb-19*
Maturity term	1.18	*2 to 7 years*
Credit rating	0.58	*BB- to B*
Call option	0.36	
Currency	0.22	*Decrease in EUR-to-USD discount between 30-Jul-18 and 14-Feb-19*
Fully adjusted interest rate	**4.28**	



E.8 Example D: sensitivity analysis

Objective: The purpose of the sensitivity analysis tool is to estimate how the interest rates depend on the change in the credit rating and maturity term parameters of the tested transaction. The analysis is typically performed as of specific valuation date and estimates as output an array of interest rates for a range of credit rating and maturity term parameters.

E.9 Validation of the IRB tool output

Appendix F AC.finance.yts Tool

The tool can be used to estimate both static and dynamic term structures. As discussed in Section 2.2, term structure estimation is an integral part of interest benchmarking analysis which does not rely on yield curve estimation by Bloomberg or Reuters.

Depending on the objectives of the term structure estimation, the tool offers several templates ranging from basic term structure model for a single credit rating to a complex dynamic model of term structures curves estimated for multiple credit ratings.

More complex models are built on top of simpler models by adding additional modelling structure as discussed below. The progression of modeling steps from simple to more complex model can be summarized as follows.

1. Term structure for a single credit rating. The modelling approach is summarized in Appendices B.3.3.1 and B.3.3.2. The approach is typically applied to estimate CCC+ or lower yield term structure that would be consistent with the exogenously given B- yield term structure (Bloomberg or Reuters B- rated yield term structure is applied as an exogenous constraint). The model uses standard equations of the NSM model to construct the yield term structure and adds a set of constraints on the estimated term structure parameters.
2. Term structure for multiple credit ratings. The modelling approach is summarized in Appendix B.3.3.3. The model estimates the yield term structures for all credit ratings endogenously (without using any exogenous yield term structure constraints). The constraints on the risk premium are modified respectively as discussed in Appendix B.3.3.3.
3. Dynamic term structure for multiple credit ratings.[12] The modelling approach is summarized in Appendix B.3.4. The model adds a dynamic component to the previous model. The tool can be applied to model yield series (as an alternative to Bloomberg or Reuters yield series).

Each static type of the term structure can be generated using the same function signature

```
ac.finance.yts.scalc(tickers[], yields[], tenors[], ratings[],
                     constraints, parameters)
```

and the dynamic type of term structure can be generated using the same function signature

```
ac.finance.yts.dcalc(tickers[], yields[][], tenors[][], ratings[],
                     constraints, parameters)
```

where the inputs and output of the function are presented below.

F.1 Static term structure for a single credit rating

Static term structure is estimated as of specific date for a specific credit rating. The section describes how to set up the template and presents the template output. The results of the estimation procedure are illustrated by an example.

1. List of tickers (ids)

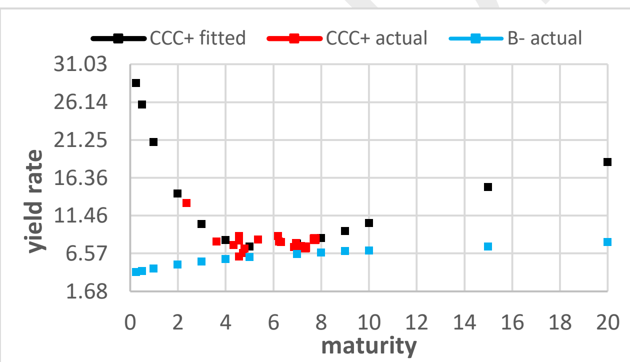
[12] The first two templates estimate static yield term structure for a single selected valuation date.

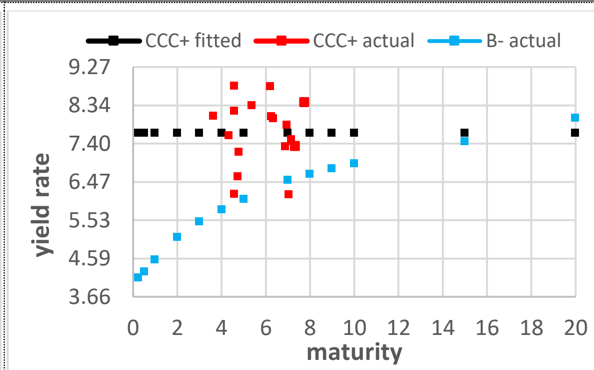
2. List of yield rates (all yields are assumed to be option-free and have the same credit rating).
3. List of tenors.
4. List of credit ratings (alternatively, list of numeric credit scores which increase with deteriorating credit rating). For models, which estimate the yield curves for a single credit rating, the parameter can be selected as empty.
5. List of weights (applied to the individual yield observations in the regression estimation).
6. List of constraints.
   - constraint-slope – a double value that specifies a ≥ constraint on the term structure slope (from below);
   - constraint-curvature-top – a double value that specifies a ≤ constraint on the term structure curvature (from above);
   - constraint-curvature-top – a double value that specifies a ≥ constraint on the term structure curvature (from below).
   - constraint-term-premium-bottom – a double value that specifies a constraint on the term premium slope from below.
7. NSM model parameters specified as a mapping with the following key/value pairs:
   - method = {Diebold-Li, Fabozzi, Nelson-Siegel, Ridge-OLS} – specifies the term structure estimation method;
   - screen-count – an integer value that specifies the number of observations (with the largest absolute values of residuals that are removed from the sample;

The results of the curve estimation for a single credit rating are illustrated in the exhibit below.

Exhibit F.1 NSM output for a single credit rating term structure curve

No constraints	Non-negative constraints on slope and curvature





The following observations can be made based on the exhibit above.

1. If the sample search strategy limits the maturity terms of the transactions, the slope and curvature become highly sensitive to small changes in the sample and can generally produce a shape with large negative slope and negative curvature (as shown in the left panel). It's normally nor recommended to (i) limit the maturity term of the notes used in the term structure estimation; and (ii) use these unconstrained estimates of the term structure with irregular shapes.

2. Parameters of the NSM model may be highly sensitive to the changes in the underlying sample. The approximation is usually robust for the maturities represented in the sample but may be highly volatile for the maturities which are not represented in the sample.
3. The term structure, presented on the right panel, was estimated by setting non-negative constraint on the term structure slope and curvature. Both constraints are binding producing a straight-line term structure.

F.2 Static term structure for a multiple credit rating

The multiple credit rating option is used in the tool in different scenario: (i) estimate a collection of term structures for multiple credit ratings; (ii) estimate a CCC+ rated term structure constrained from above by exogenously given B+ term structure; or (iii) estimate CCC and CCC+ rated term structures constrained from above by exogenously given B+ term structure.

The CCC+ rated term structure (constrained from the above) is estimated using multi credit rating option of the tool, to ensure not only a positive spread between the CCC+ and B- rated term structures, but also consistency in the shapes between the two term structures. For a similar reason, the CCC and CCC+ rated term structures are estimated using the tool option.

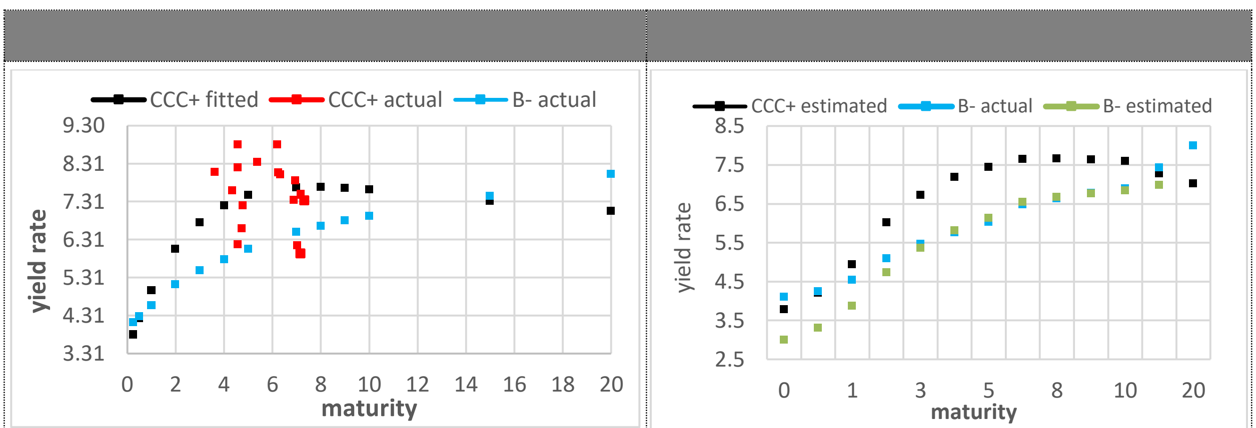
The calculator uses the following inputs.

Inputs 1 – 5, 7 are the same as in the previous template.

6. List of constraints (in addition to the constraints from the previous template, the following additional constraints can be applied).
   - constraint-slope-constant – a 0/1 value which specifies whether a constant slope is applied for different credit ratings. Default value is 0 (slope is different for different credit ratings);
   - constraint-curvature-constant – a 0/1 value which specifies whether a constant curvature is applied for different credit ratings. Default value is 1 (curvature is the same for different credit ratings).

The credit rating constraints are generated automatically by the term structure estimation tool based on the provided credit rating (credit score) inputs. However, exogenous constraints on the modelled yield series from above and below can still be included.

Exhibit F.2 NSM output for a CCC+ rated term structure

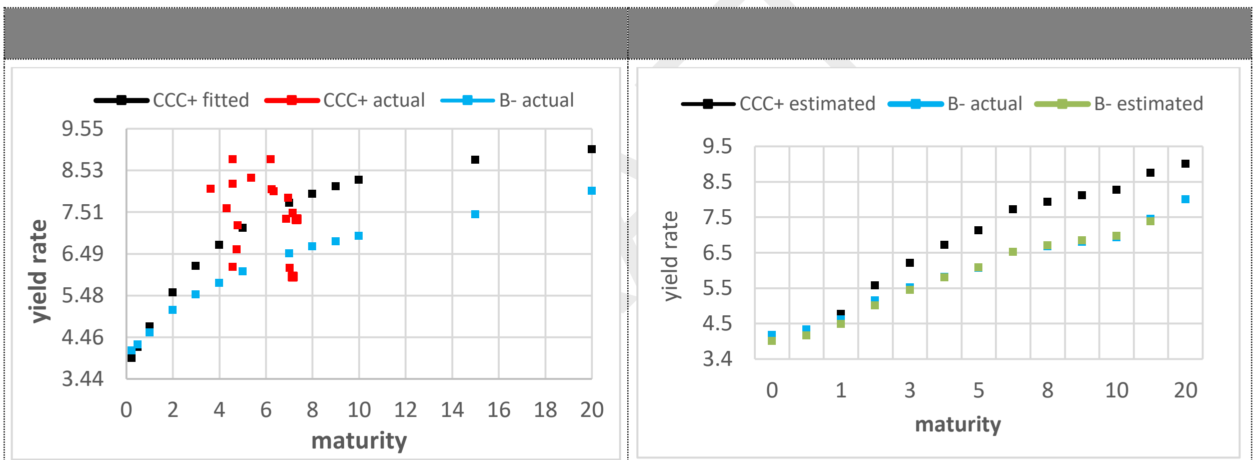


The exhibit above shows that including an exogenous B- term structure as part of estimation model imposes certain constraints on the estimated CCC+ term structure so that the have consistent shapes. In the above example, however, the estimate of the B- term structure does not match closely the actual exogenous B- term structure for short-term maturities. Therefore, to improve consistency between the CCC+ rated and exogenously given B- term structure, the matching of the short-term B- rated term structure needs to be improved.

In addition, the exhibit shows a high positive curvature of the estimated CCC+ term structure, which represents itself with a steep increase from short-term to medium-term rates and a decrease from medium-term to long-term yields. This abnormal share can often be remedied by setting a non-negative term structure slope constraint (constraint-term-premium-bottom ≥ 0). The constraint straightens the long-term rates by reducing the slope curvature.

To improve the matching of a specific part of the term structure, we assign larger weights to the selected observations (in this case, weight 10.0 was assigned to B- part of the term structure with maturities ranging from 0.25 to 2.0. The default weight assigned to each observation is 1.0). The results of the term structure estimation with tuned parameters is illustrated in the exhibit below.

Exhibit F.3 NSM output for a CCC+ rated term structure



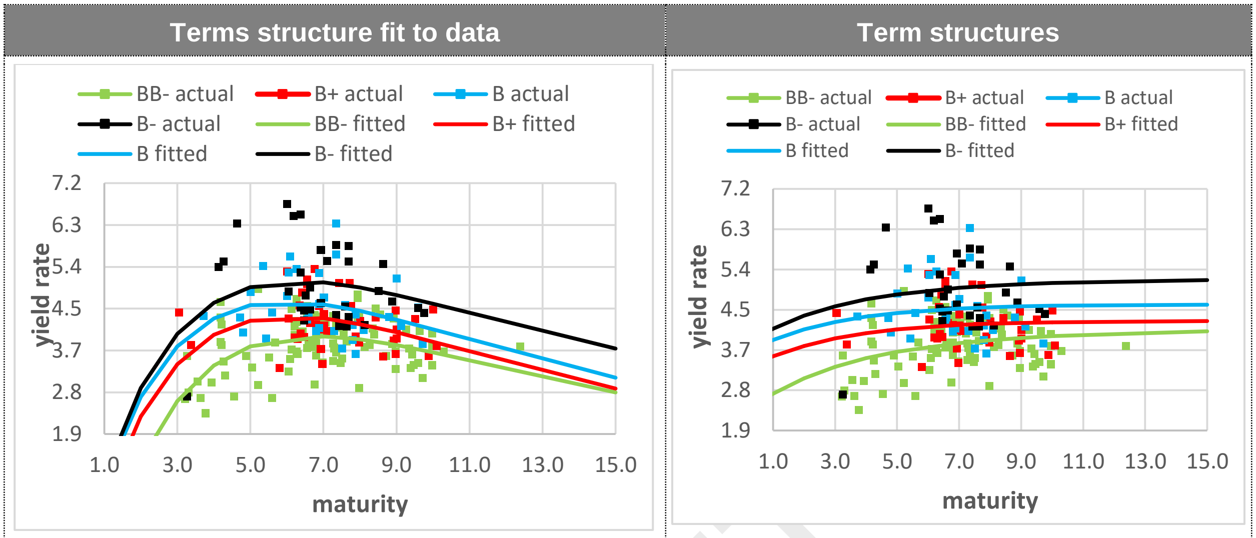
The following observations can be made based on the exhibit above.

1. The weights inputs should be tuned in the tool to match closely the exogenously given term structure constraints.
2. Simultaneous estimation of the term structures for multiple ratings improves consistency in the term structure shapes.
3. The consistency in the term structure shapes can be further improved by setting slope / curvature constant for the curves. By default, the slope is assumed to be different and the curvature is assumed to be constant over the term structures with different credit ratings.
4. The minimum spread can be set as a parameter in the tool. The minimum spread can be set either at 0, at 2, or at 25 basis points.

The next example below demonstrates how the tool can be used to estimate a collection of term structures for several credit ratings. The term structures were estimated for the B to BB- rated US$ yields obtained through Reuters database. The search was limited to the North American region and broad industrial sector (excluding sectors affected by COVID, such as leisure, transportation, retail, and others).

The term structures were estimated assuming non-negative slope constraint and assuming minimum 5bps premium between B and B+ yield rates.

Exhibit F.4 NSM term structure output for multiple credit ratings



The following observations can be made based on the exhibit above.

1. The data shows downward curvature in the yields. The pattern is represented by a large curvature coefficient of the term structure series. The curvature results in a sharp increase in the yields for short-term maturities and a downward slope for long-term maturities. The term structure pattern outside the yield data may not be reliable.
2. To remedy the curvature problem, the non-negative term structure slope constraint is applied to the curves.

The impact of the 'non-negative term structure' constraint is shown in the complementary slackness conditions of the quadratic optimization problem. The Lagrange multipliers (shown in the second column in the exhibit below) are positive for the 15 to 20 term premia of the B and B+ curves, implying that the constraints are binding. To ensure that the constraints are satisfied, the algorithm reduces the curvature parameter to straighten the curves. As a result, a curve is transformed into a regular-looking term structure curve.

term premium[BB-, t=(15.0, 20.0)] >= 0.0	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00	0.01
term premium[B+, t=(15.0, 20.0)] >= 0.0	0.11	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	-0.00
term premium[B, t=(15.0, 20.0)] >= 0.0	0.15	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	0.00
term premium[B-, t=(15.0, 20.0)] >= 0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00
term premium[(BB-, B+), t=]0.25] >= 0.0	0.00	-1.00	1.00	0.00	0.00	-0.96	0.96	0.00	0.00	0.00	0.00	0.91

F.3 Dynamic term structure

The calculator uses the following inputs.

1. List of tickers
2. Sample of yield rates.
3. Sample of tenors.

4. List or sample of credit ratings (or credit scores).
5. List of constraints
6. Mapping with NSM model parameters.

F.4 Term structure validation

Since the estimation procedure relies significantly on the quadratic optimization problem, the tool includes the validation equations for the optimization problem first-order conditions. Specifically, the following two first-order condition equations are validated.

(i) First-order derivative equation: $\mathrm{f} \times \Lambda^T \Lambda - (\Lambda \mathrm{y})^T = \alpha \times \mathrm{C}$

(ii) Complementary slackness condition: $\alpha \circ (Cf - c) = 0$

(iii) Constraints: $\alpha \geq 0$ and $Cf - c \geq 0$.

where ° is the dot product of two vectors.

F.5 Technical notes

This section provides a technical description of the tool implementation.

F.5.1 Model structure

The term structure calculator is implemented using a separate calculator class for different model type: calculator which estimates (i) term structure for a single rating; (ii) term structure for multiple ratings; and (iii) dynamic term structure. The dynamic term structure calculator is implemented as a collection of static term structure calculators.

All calculators extend an abstract ‘basic calculator NSM’ (which in turn extends abstract ‘calculator NSM’ class). The basic calculator implements the core functionality, such as (i) estimation of constraint regression coefficients; (ii) estimation of the non-linear alpha parameter; (iii) outlier identification and removal; and other. The calculators which estimate a specific term structure model (‘calculator single curve NSM’, ‘calculator multi curve NSM’, and ‘calculator dynamic NSM’) implement functionality which is specific to each model.

The calculator classes are instantiated through a factory class, which converts inputs provided by a user into the respective calculator objects.

F.5.2 Modelling constraints

On the user side, the constraints are modelled through a mapping with the following key, value pairs.

1. Key = ‘constraint-slope’. Respective value is a number, which constraints the slope from below.
2. Key = ‘constraint-curvature-top’. Respective value is a number, which constraints the slope from above.
3. Key = ‘constraint-curvature-bottom’. Respective value is a number, which constraints the slope from below

4. Key = ‘constraint-array’. Respective value is a range, which models a left-hand-side matrix with additional custom constraints on the regression coefficients.
5. Key = ‘constraint-vector’. Respective value is a range, which models a right-hand-side vector with additional custom constraints on the regression coefficients.

In addition, there are the following constraints on the set of modelled term structure coefficients.

1. Key = ‘constraint-slope-constant’. Respective value is either 0 or 1. If the value is 1, then the same slope is modelled for each credit rating.
2. Key = ‘constraint-curvature-constant’. Respective value is either 0 or 1. If the value is 1, then the same curvature is modelled for each credit rating.

The constraints are converted into the matrix form $C \times f \geq c$. Specifically,

1. In the case of the ‘constraint-yield-curve-top’ constraint, each (τ, y_τ^*) pair is converted into the following constraint: $-\Lambda_\tau^* \times f \geq -y_\tau^*$.
2. In the case of the ‘constraint-yield-curve-bottom’ constraint, each $(\tau, y_{\tau,*})$ pair is converted into the following constraint: $\Lambda_{\tau,*} \times f \geq y_{\tau,*}$.
3. In the case of the ‘constraint-slope’ constraint, the following constraint is added to the set of constraints: $(0,1,0) \times f \geq c^{slope}$.
4. In the case of the ‘constraint-curvature’ constraint, the following constraint is added to the set of constraints: $(0,0,1) \times f \geq c^{curvature}$.

The ‘constraint-slope-constant’ and the ‘constraint-curvature-constant’ are applied in the case of the term structure modelling for multiple ratings. The constraints determine the structure of the linear regression model.

The generated set of constraints is passed then as inputs into the class which models and estimates linear regression with linear inequality constraints, which is represented as the constraint quadratic optimization problem.

The class, which implements solution of the constraint quadratic optimization problem, is based on the free open-source solver[13], which applies Goldfarb – Idnani (1983) algorithm.[14] The solver models the minimization problem using the $C^T \times f \geq c$ matrix constraint format. Therefore, the constraint matrix C is transposed. The conversion to the proper format is done internally as the solver class is instantiated. If the user includes any customized constraints in the array form, the format used in this guide should be followed ($C \times f \geq c$).

The model of a single credit rating term structure, which is constrained from above / below by a given (Blomberg) term structure, is reduced to a term structure model with multiple credit ratings. The exogenous term structures are added to the sample. The approach is convenient as it also allows to impose the slope / curvature constraints to produce the shape of the custom term structure which is consistent with the slope / curvature of the exogenous term structure inputs.

In the model with the term structures estimated for multiple credit ratings, the sample data is sorted in increasing order with respect to deteriorating credit rating. The model parameters assume respectively the

[13] http://www2.econ.iastate.edu/tesfatsi/DCOPFJHome.htm#GI83.

[14] Donald Goldfarb and Ashok Udhawdas Idnani (1983), "A Numerically Stable Dual Method for Solving Strictly Convex Quadratic Programs", Mathematical Programming, Vo. 27, pp. 1-33.

following order: f_0 parameters for deteriorating credit ratings; f_1 parameters deteriorating credit ratings (or a single f_1 parameter if the slope is constant); and f_2 parameters deteriorating credit ratings (or a single f_2 parameter if the curvature is constant).

F.5.3 Modelling outlier removal

The tool removes automatically a subset of yields, which are interpreted as outliers. The number of eliminated yields is controlled by a 'screen-count' key of the parameter mapping. The outlier removal is implemented using the following algorithm.

1. Identify the observations with the largest deviation from the estimated term structure (or term structures if they are estimated for multiple credit ratings).
2. Remove the observation and re-estimate the term structure(s).
3. Continue steps 1 – 2 until the specified number of observations are removed from the sample.

Note that the algorithm does not depend on which yield term structure scenario is being estimated.

F.6 Summary

The following default parameters are recommended for the tool:

1. Default constraints
   - constraint-term-premium-bottom ≥ 0
   - constraint-slope-constant = 0
   - constraint-curvature-constant = 1
2. Default parameters
   - method = Fabozzi[15]
   - screen-count – based on the observed outlier data

[15] Fabozzi alpha = 0.333 parameter was selected as it produced a more regular shape of the term structure curves.

Appendix G Examples

xx

G.1 Forward term structure estimation

An example of forward term structure estimation performed based expectation and liquidity premium theories is illustrated below.

G.1.1 Estimation based on expectation theory

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G.1.2 Estimation based on liquidity theory

Appendix H References

The list of literature references used in this guide is provided below.

[1] Bolder D and D Stréliski, Yield Curve Modelling at the Bank of Canada, Technical Report No. 84, Bank of Canada, February 1999

[2] Diebold FX, GD Rudebusch and SB Aruoba, The Macroeconomy and the Yield Curve: a Dynamic Latent Factor Approach, Journal of Econometrics 131 (2006), pp. 309-338

[3] Hull J., White A., “Libor vs. OIS: The Derivative Discounting Dilemma”, *Journal of Investment Management*, Vol. 11, No. 3, pp.14-27

[4] “OECD Transfer Pricing Guidelines for Multinational Enterprises and Tax Administrations”, OECD, July 2017

[5] “Transfer Pricing Guidance on Financial Transactions”, Inclusive Framework on BEPS: Actions 4, 8–10, February 2020, OECD

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